Dual Bose-Fermi Superfluids

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The goals of quantum simulation

- Obtain results on a quantum system that cannot be reached by standard methods or numerical simulations
- Explore novel geometries, parameters, or configurations that are not available in the initial system
- Invent novel situations or devices based on the acquired knowledge

Cold atoms are good quantum simulators

Non-trivial questions:

- How to verify the simulation results?
- How to detect and correct errors?
106 years of quantum fluids

Bose Einstein condensate

$^4$He

T~ 2.2 K

Superconductivity

High $T_c$

77 K

$^3$He

2.5 mK

100 nK

BCS-BEC Crossover

dilute gas BEC

+ polaritons and BEC of light

Fermi gas superfluid
Outline

• Equation of state of fermions with tunable interaction

• Dual Bose-Fermi superfluid recipe

• The critical velocity for superfluid Bose-Fermi counterflow

• Lifetime of the Bose Fermi mixture: a simple formula!


3) Y. Castin, I. Ferrier-Barbut and C. Salomon

4) S. Laurent, M. Pierce, M. Delehaye, T. Yefsah, F. Chevy, C. Salomon

5) M. Abad, A. Recati, S. Stringari, F. Chevy, EPJD, 69, 2015

6) P-P. Crépin, X. Leyronas, F. Chevy, ArXiv:1607.00218
Searching for superfluid Bose-Fermi systems: $^4\text{He} - ^3\text{He}$ mixture

Volovik, Mineev, Khalatnikov, JETP, 42, 342 (1975): Fermi liquid theory of mixture

*Expected $T_c \sim 1$ to $20 \mu K$?*
$^6\text{Li}$ and $^7\text{Li}$ isotopes

$^6\text{Li}$ (fermion)

$^7\text{Li}$ (boson)
Equation of State of Fermi gas in the BEC-BCS crossover

Pressure equation of state \[ \frac{P}{P_0} = f\left(\frac{1}{k_F a}\right) \]

An example of quantum simulation in the strongly correlated regime

Bose-Fermi superfluidity recipe

Requirements:
- Low $a_{bf}$ (no interspecies demixing)
- High $|a_f|$ (high fermionic superfluid $T_c$)
- Positive $a_{bb}$ (stable BEC)

$^6\text{Li} - ^7\text{Li}$ mixture in the $|1\rangle_f$, $|2\rangle_f$ and $|2\rangle_b$

$a_{bf} = 40.8 \ a_0$
Unitary $^6$Li Fermi gas can cool any species fulfilling the requirements to BEC.
See also $^6$Li-$^{41}$K, USTC, China, PRL ’16, and $^6$Li-$^{173}$Yb, UWash, PRL’17.

Cool molecules to quantum regime?
Long-lived Oscillations of both Superfluids

Fermi Superfluid

\[ \tilde{\omega}_6 = 2\pi \times 17.06(1) \text{Hz} \]
\[ \tilde{\omega}_7 = 2\pi \times 15.40(1) \text{Hz} \]

Coupled Superfluids

Single Superfluid

Ratio = \((7/6)^{1/2} = (m_7/m_6)^{1/2}\)
Oscillations of both superfluids

Very small damping!
Modulation of the $^7$Li BEC amplitude by $\sim$30% at

$$\left(\tilde{\omega}_6 - \tilde{\omega}_7\right)/2\pi$$

Coherent energy exchange between the two oscillators
Dual Bose-Fermi superfluids with $^6\text{Li}-^7\text{Li}$ isotopes

**Fermi Superfluid**

**Question 1:** How to understand the oscillation frequencies?

**Question 2:** What is the critical velocity for superfluid counterflow?

**Question 3:** What is the lifetime of the Bose-Fermi mixture?

At unitarity, the lifetime is 7 seconds in shallow optical trap. How does it vary with $1/k_f a_f$, with $a_{bf}$, and with density?
1.5% down shift in $^7$Li BEC frequency

BEC osc. amplitude beat at frequency  $(\tilde{\omega}_6 - \tilde{\omega}_7) / 2\pi$

Weak interaction regime: $k_F a_{bf} << 1$ and $N_7 << N_6$

Boson effective potential  \[ V_{eff} = V(r) + g_{bf} n_6(r) \quad \text{with} \quad g_{bf} = \frac{2\pi \hbar^2 a_{bf}}{m_{67}} \]

\[ m_{67} = m_6 m_7 / (m_6 + m_7) \]

LDA  \[ n_6(r) = n_6^0 (\mu_6^0 - V(r)) \]

Where  \[ n_6(\mu) \] is the Eq. of State of the stationary Fermi gas.

For the small BEC:  \[ V(r) << \mu_6^0 \]

Expand  \[ n_6(r) \approx n_6^0 (\mu_6^0) - V(r) \frac{dn_6^0}{d\mu_6} + ... \]
Boson effective potential and link with Equation of State

Thomas Fermi radius of BEC<< TF radius of Fermi Superfluid:

\[ V_{\text{eff}} = g_{bf} n_{6}(0) + V(r) \left[ 1 - g_{bf} \left( \frac{dn_{6}^{(0)}}{d \mu_{6}} \right)_{0} \right] \]

The potential remains harmonic with rescaled frequency

\[ \tilde{\omega}_{7} = \omega_{7} \sqrt{1 - g_{bf} \left( \frac{dn^{(0)}}{d \mu_{6}} \right)_{0}} \]

A new means to access properties of the EoS!

The equation of state \( n(\mu) \) at low T is known in the BEC-BCS crossover N. Navon et al., Science, 2010

Example: at unitarity, \( 1/a = 0 \)

From Thomas Fermi radius of \(^6\text{Li} \) superfluid, we find: \( \tilde{\omega}_{7} = 2\pi \times 15.43 \text{ Hz} \)

very close to the measured value:

\( \tilde{\omega}_{7} = 2\pi \times 15.40(1) \text{ Hz} \)
Equation of State and Bose-Fermi Coupling in BEC-BCS crossover

\[ \frac{\delta \tilde{\omega}}{\omega} k_F a_{bf} \approx 6.190 \frac{a_{bf}}{a_f} \]

From EoS in the crossover
N. Navon et al, Science 2010

Shift in BEC limit

MIT '12

BCS

NIFG
What is the critical velocity for superfluid counterflow?

- Increase initial displacement
- Increase relative velocity
Critical velocity for superfluid counterflow

\[ d = d_0 \exp(-\gamma t) + d' \]

\[ \gamma = 3.1 \, s^{-1} \]

Time (ms)

Initial damping

\[ V_c = 2 \, \text{cm/s} \]

is quite high!
Momentum Conservation: \[ MV = MV' + \hbar \mathbf{k} \]

Energy Conservation: \[ MV^2 / 2 = MV'^2 / 2 + \epsilon_k \]

\[ \hbar k V \geq \hbar \mathbf{k} \cdot \mathbf{V} = \epsilon_k + \hbar^2 k^2 / 2M \geq \epsilon_k \]

Motion of impurity is damped by the creation of elementary excitations if:

\[ V \geq V_c = \min_k \left( \frac{\epsilon_k}{\hbar k} \right) \]

For a linear excitation spectrum \( \epsilon_k = \hbar kc \), \( V_c = c \), the sound velocity.
Critical velocities
Landau criterion for a Bose-Fermi mixture @ T=0

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\[ E_{B,k} = \varepsilon_{B,k} + \hbar k \cdot V_B \]

\[ E_{F,k'} = \varepsilon_{F,k'} + \hbar k' \cdot V_F \]

Energy-momentum conservation:

\[ E_{B,k} + E_{F,k'} = 0 \quad k + k' = 0 \]

\[ |V_B - V_F| \geq \min_k \left( \frac{\varepsilon_{B,k} + \varepsilon_{F,-k}}{\hbar k} \right) \]

Sound Modes:

\[ V_c = c_B + c_F \]

Counterflow critical velocity

\[ \gamma(s^{-1}) \]

\[ v_{\text{max}}/v_F \]

- \( c_B \)
- \( c_F \)
- \( c_B + c_F \)
Question 3:
What is the lifetime of the Bose-Fermi mixture?

Three-body recombination as a probe of quantum correlations in a strongly interacting system.
Three-body recombination in Bose-Fermi mixture

As $a_{bf}$ is small, bosons act as a weakly coupled impurity immersed in a Fermi gas with large $a_f$

Three-body recombination: $i, \downarrow, \uparrow$

Decay to a deeply bound molecular state
Binding energy transferred to kinetic energy of collision partners
Atom and molecule leave the trap
A weakly coupled impurity in a resonant Fermi gas

Unitarity: ?

Unitary regime

Assuming a saturation

\[ a \sim n_f^{-1/3} \]

We expect:

\[ \Gamma_{if} \propto n_f^{4/3} \]

BEC Side

- "Two" body
- Dimer-impurity losses

\[ \dot{n}_i = -L_{di} n_d n_i \]

\[ L_{di} \propto 1 / a_f \]

JILA: Zirbel et al., PRL 100, 143201 (2008)

BCS Side

- Three body losses

\[ \dot{n}_i = -L_{ff} n_i n_f^2 \]

\[ L_{ff} \propto a_f^2 \]

D’incao and Esry, PRL 2008
Zirbel et al., PRL 2008
Spiegelhalder et al., PRL 2009
Khramov et al., PRA 2012
A weakly coupled impurity in a resonant Fermi gas

Kagan, Svistunov, Shlyapnikov, JETP, 1985

\[ P(R < R^*) = \int_{R < R^*} d^3r_1 d^3r_2 d^3r_3 \left\langle \Psi_1^\dagger (r_1) \Psi_2^\dagger (r_2) \Psi_i^\dagger (r_3) \Psi_i (r_3) \Psi_2 (r_2) \Psi_1 (r_1) \right\rangle \]

Weak coupling between the impurity and the resonant fermions

\[ P(R < R^*) = \int_{R < R^*} d^3r_1 d^3r_2 d^3r_3 \left\langle \Psi_1^\dagger (r_1) \Psi_2^\dagger (r_2) \Psi_2 (r_2) \Psi_1 (r_1) \right\rangle \left\langle \Psi_i^\dagger (r_3) \Psi_i (r_3) \right\rangle \]

\[ g_{\uparrow \downarrow} (r_2, r_1) \sim \frac{1}{\Omega} \frac{C_2}{4\pi^2 |r_2 - r_1|^2} \]

With:

Therefore the impurity decay rate \( \Gamma_{if} \) should be proportional to Tan’s two-body contact \( C_2 \)
Tail of the momentum distribution at large $k$

$$k^4 n_\sigma(k) \rightarrow C_2 \quad \text{when } k \rightarrow \infty$$

*JILA: Stewart et al., Jin’s group, PRL, 2010*

Adiabatic energy relation

$$C_2 = -\frac{4\pi m_f}{\hbar^2} \frac{\partial E}{\partial (1/\alpha)}$$

at constant entropy

From equation of state measurements:

*ENS, Navon et al., Science, 2010*
Bose/Fermi decay and Tan’s Contact

\[ \dot{n}_b = -\gamma C_2 n_b = -\Gamma_{bf} n_b \]

\[ \gamma \propto a_{bf}^2 \]

is the only parameter that contains short range physics easily measured at high temperature on BEC side

\[ \zeta = 0.87(3) \]

C. Vale, Swinburne
Bose/Fermi decay in strongly interacting regime

\[ \dot{n}_b = -\gamma C_2 n_b = -\Gamma_{bf} n_b \]

\[ \Gamma_{bf} \propto 1/a_{ff} \]

BEC + Fermi Superfluid

\[ \Gamma_{bf} \propto n_f^{4/3} \]
Probing the local unitary Contact

\[ R_{TF,b} = 0.3 R_{TF,f} \]

Prediction with no adjustable parameter

\[ \Gamma_{bf} = \gamma C_2 = \frac{2 \zeta}{5 \pi} \left( 3 \pi^2 n_f^{4/3} \right) \times 0.9 \]

Average over BEC TF profile
Probing the local unitary Contact

Power law fit: $A n^p$ gives $p = 1.36 (15)$ close to $4/3$

Fit: $A n^{4/3}$ gives $A$ and local contact $C_2(0)$

Impurity decay is a local probe of quantum correlations in a many-body system
Summary

• Dual Bose-Fermi superfluids have intriguing novel properties

• Lifetime of Bose-Fermi mixture is governed by Tan’s contact

• Theory applies to Yb$^6$Li, K$^6$Li, Rb$^{40}$K, ….. assumes small $a_{bf}$

• What happens when $a_{bf}$ increases ? Efimov effect

• Measure three-body contact of Fermi gas

• Two-body and three-body contact in unitary Bose gas

R. Fletcher et al., Science 2017, Cambridge
Link with lifetime measured at JILA

C. E. Klauss et al., ArXiv 1704.01206