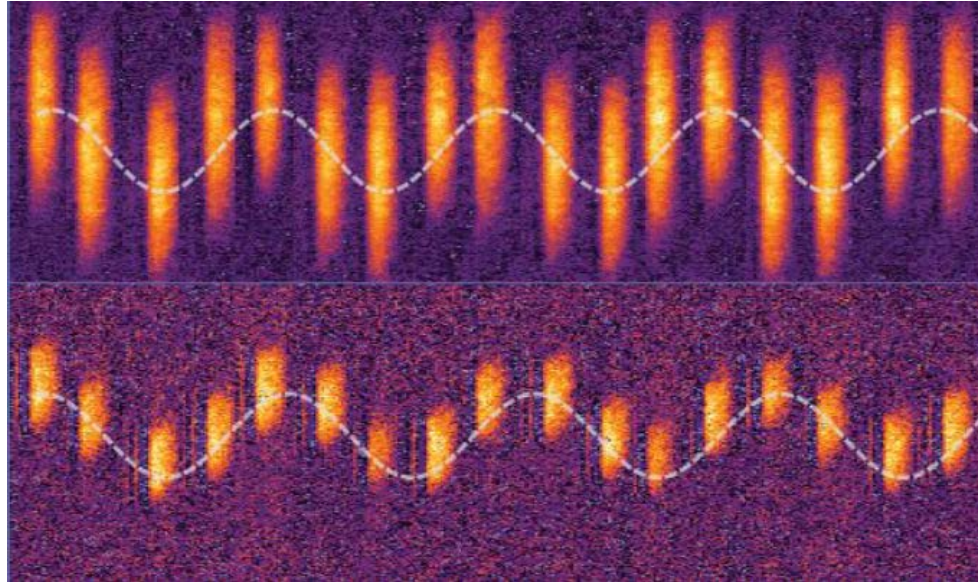


Dual Bose-Fermi Superfluids



COLLÈGE
DE FRANCE
— 1530 —

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ICQSIM, Paris, November 13-17, 2017



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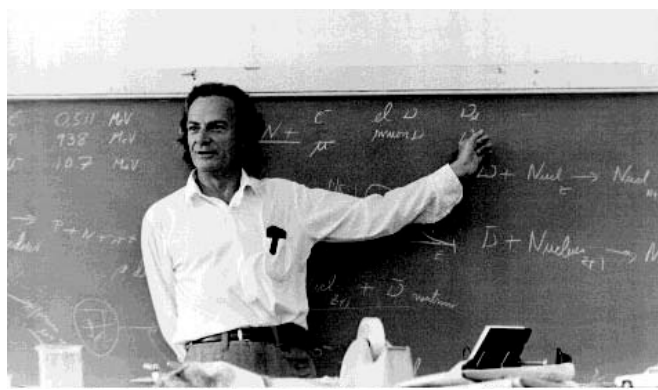


I. Ferrier-Barbut

Theory:

Y. Castin, F. Werner, X. Leyronas (ENS), S. Stringari (Trento), A. Recati, T. Ozawa, O. Goulko (Amherst), C. Lobo, J. Lau (Southampton), I. Danaila (Rouen)

The goals of quantum simulation



- ➔ Obtain results on a quantum system that cannot be reached by standard methods or numerical simulations
- ➔ Explore novel geometries, parameters, or configurations that are not available in the initial system
- ➔ Invent novel situations or devices based on the acquired knowledge

Cold atoms are good quantum simulators

Non-trivial questions:

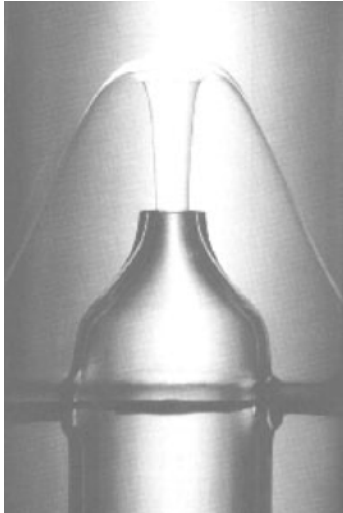
How to verify the simulation results ?

How to detect and correct errors ?

106 years of quantum fluids

Bose Einstein condensate

^4He



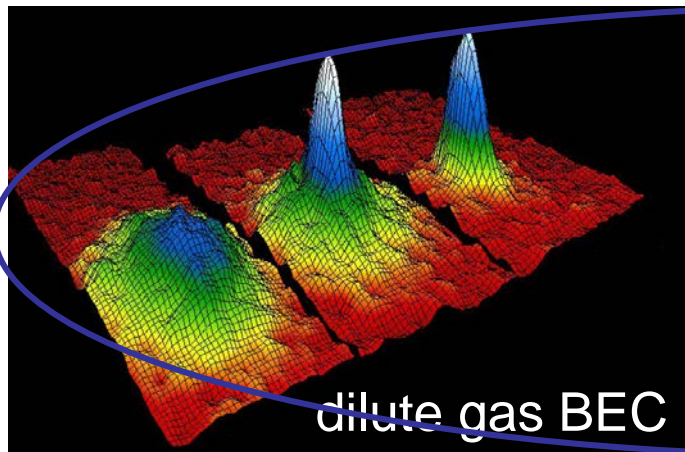
$T \sim 2.2 \text{ K}$

Superconductivity

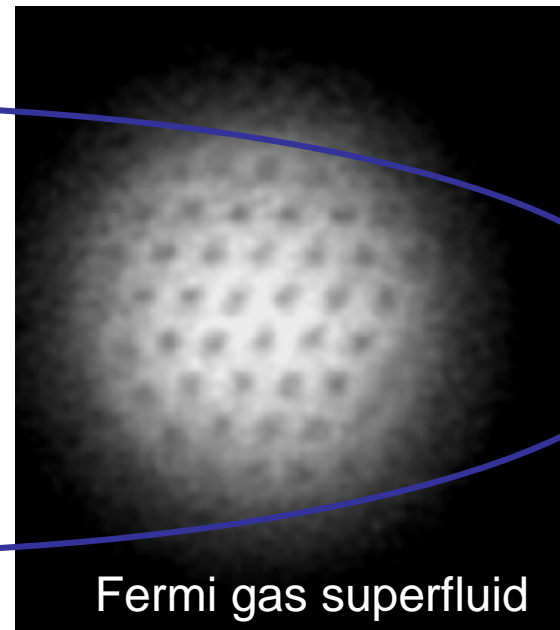
High T_c
77 K



^3He
2.5 mK



100 nK
BCS-BEC
Crossover



+ polaritons and BEC of light

Outline

- Equation of state of fermions with tunable interaction
- Dual Bose-Fermi superfluid recipe
- The critical velocity for superfluid Bose-Fermi counterflow
- Lifetime of the Bose Fermi mixture: a simple formula !

1) I. Ferrier-Barbut, M. Delehaye, S. Laurent, A. T. Grier, M. Pierce, B. S. Rem, F. Chevy, and C. Salomon, *Science*, **345**, 1035, 2014

2) M. Delehaye, S. Laurent, I. Ferrier-Barbut, S. Jin, F. Chevy, C. Salomon, *PRL*, **115**, 265303, 2015

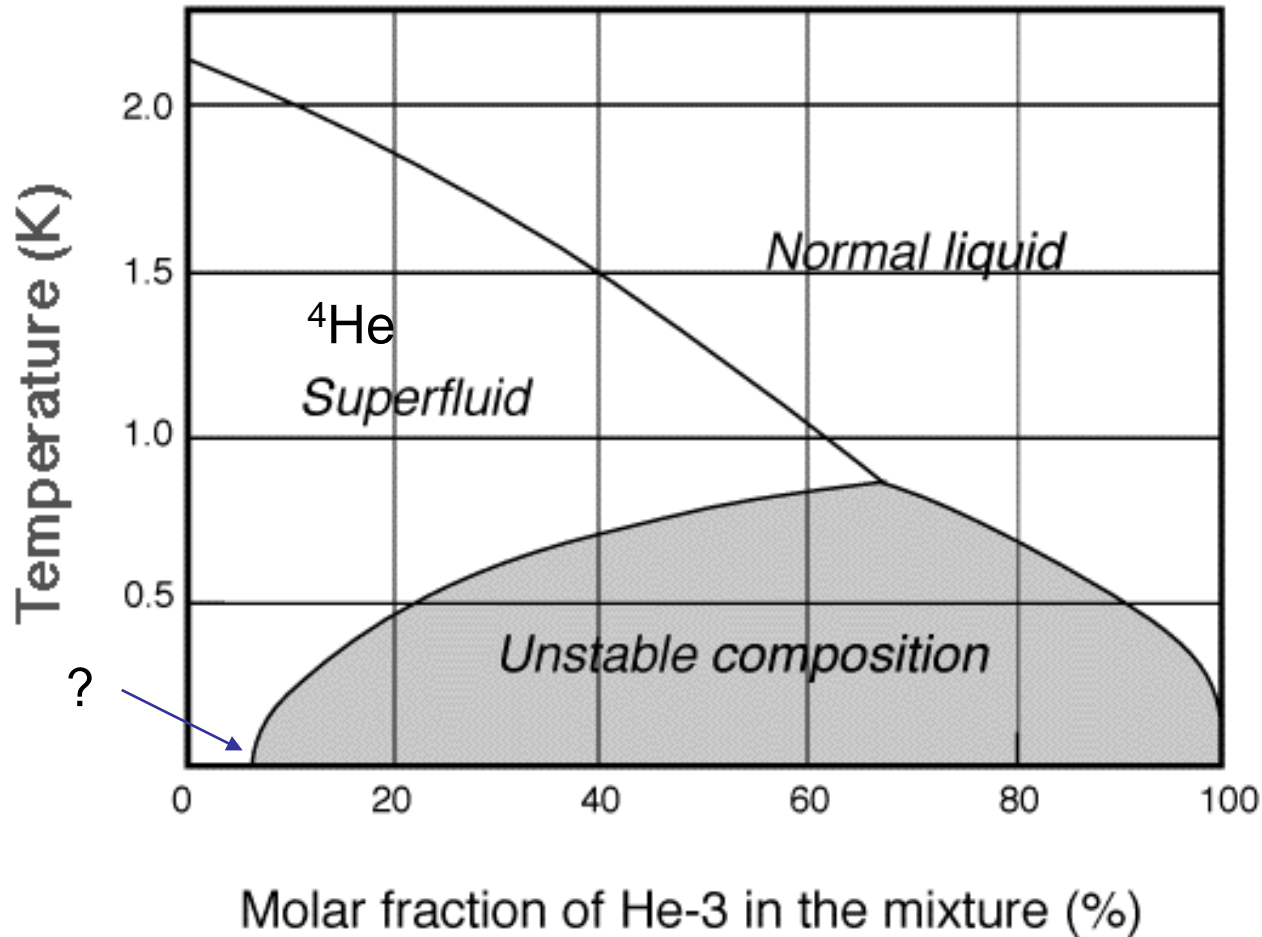
3) Y. Castin, I. Ferrier-Barbut and C. Salomon
Comptes-Rendus Acad. Sciences, Paris, **16**, 241, 2015

4) S. Laurent, M. Pierce, M. Delehaye, T. Yefsah, F. Chevy, C. Salomon
Phys. Rev. Lett., **118**, 103403, 2017

5) M. Abad, A. Recati, S. Stringari, F. Chevy, *EPJD*, **69**, 2015

6) P-P. Crépin, X. Leyronas, F. Chevy, *ArXiv*:1607.00218

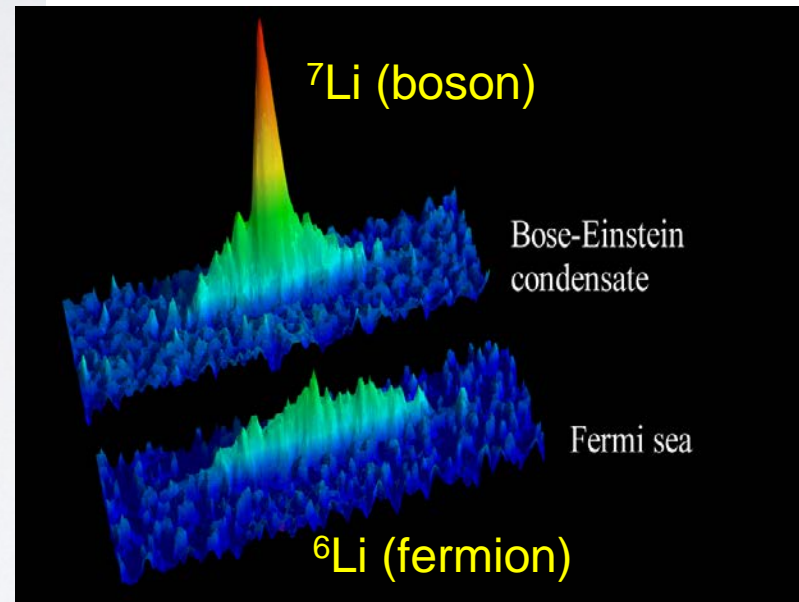
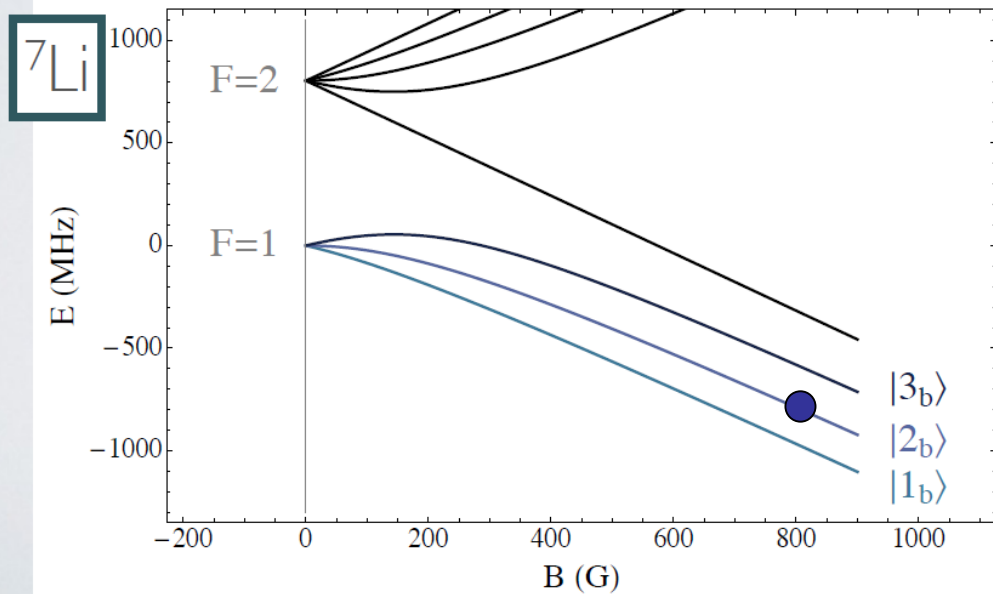
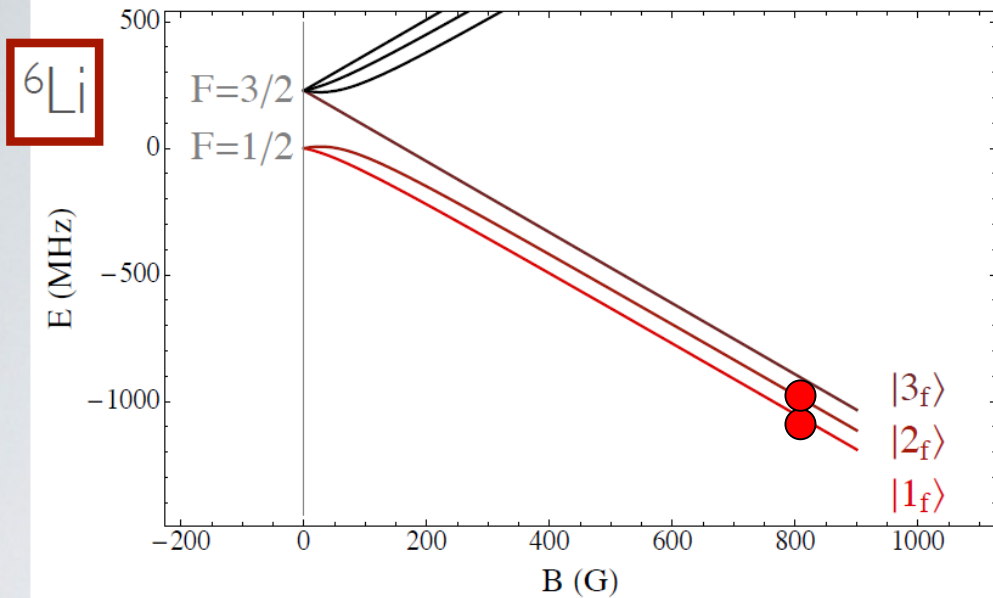
Searching for superfluid Bose-Fermi systems: ^4He - ^3He mixture



Volovik, Mineev, Khalatnikov, JETP, 42, 342 (1975): Fermi liquid theory of mixture

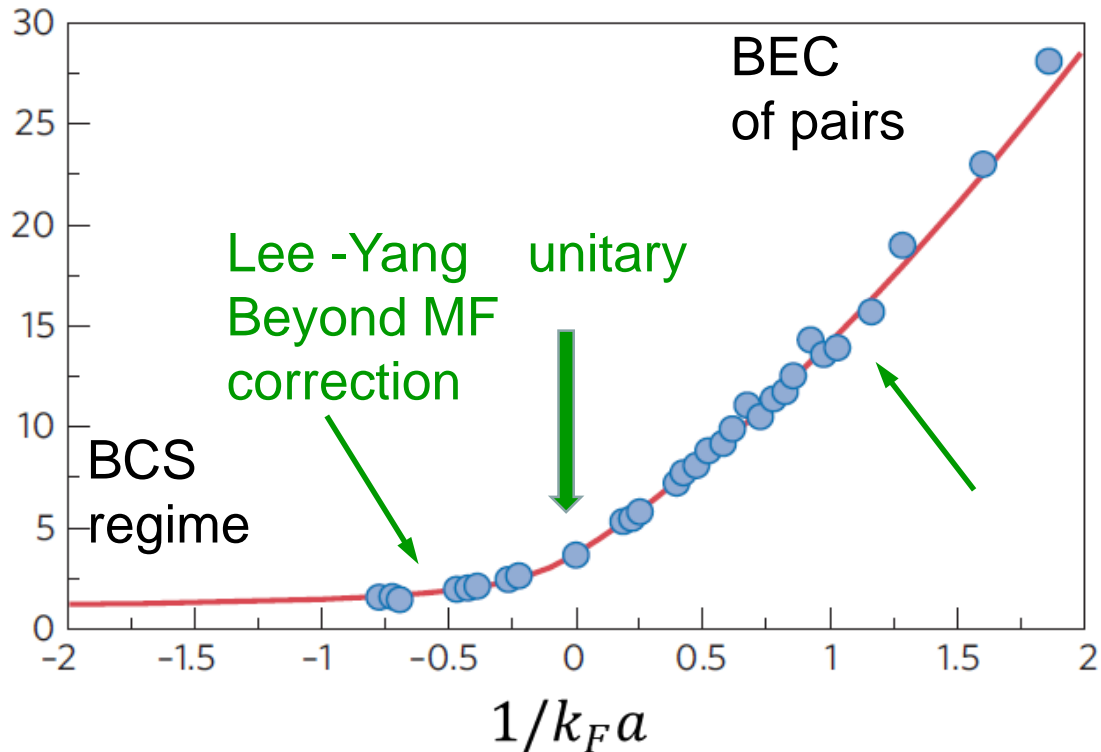
Expected $T_c \sim 1$ to $20 \mu\text{K}$?

${}^6\text{Li}$ and ${}^7\text{Li}$ isotopes



Equation of State of Fermi gas in the BEC-BCS crossover

Pressure equation of state $P/P_0 = f(1/k_F a)$



BCS-BEC crossover
at $T \sim 0$

Lee Huang Yang
Beyond MF
correction

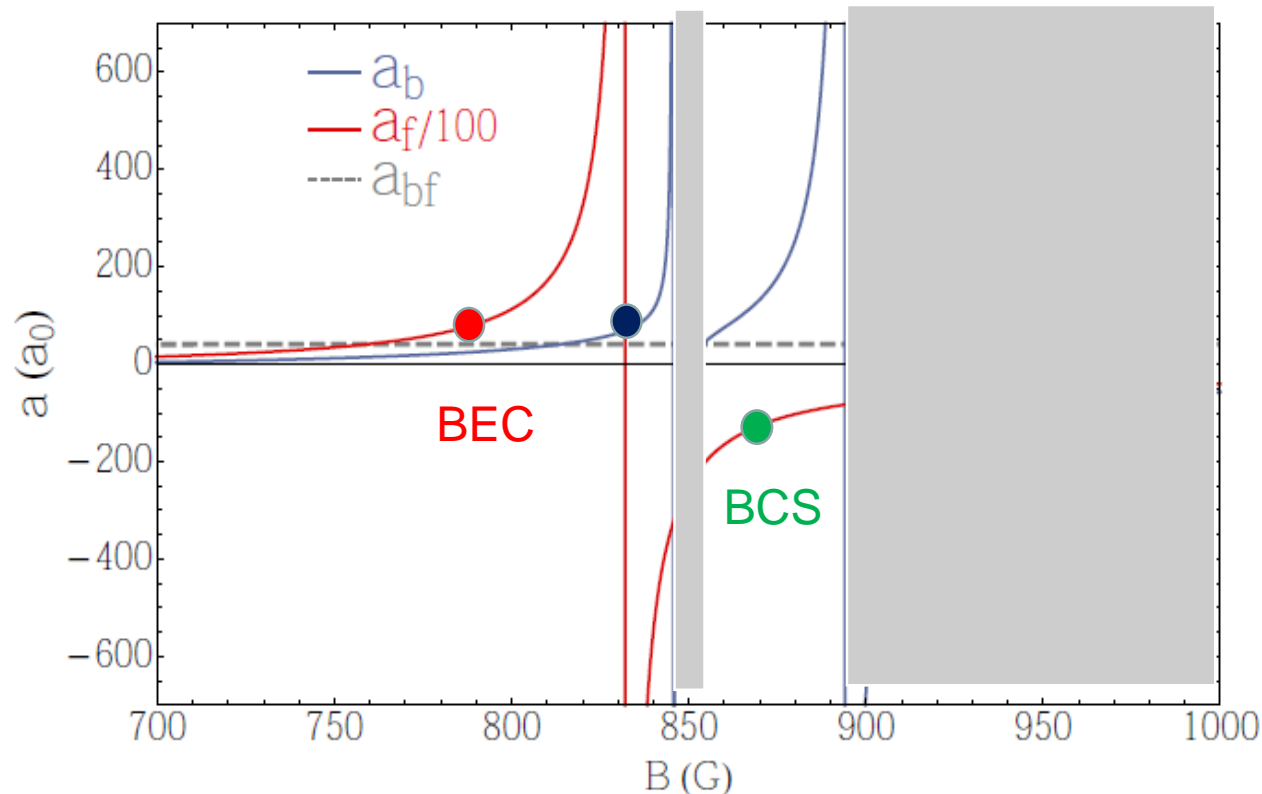
An example of quantum simulation in the strongly correlated regime

Bose-Fermi superfluidity recipe

Requirements:

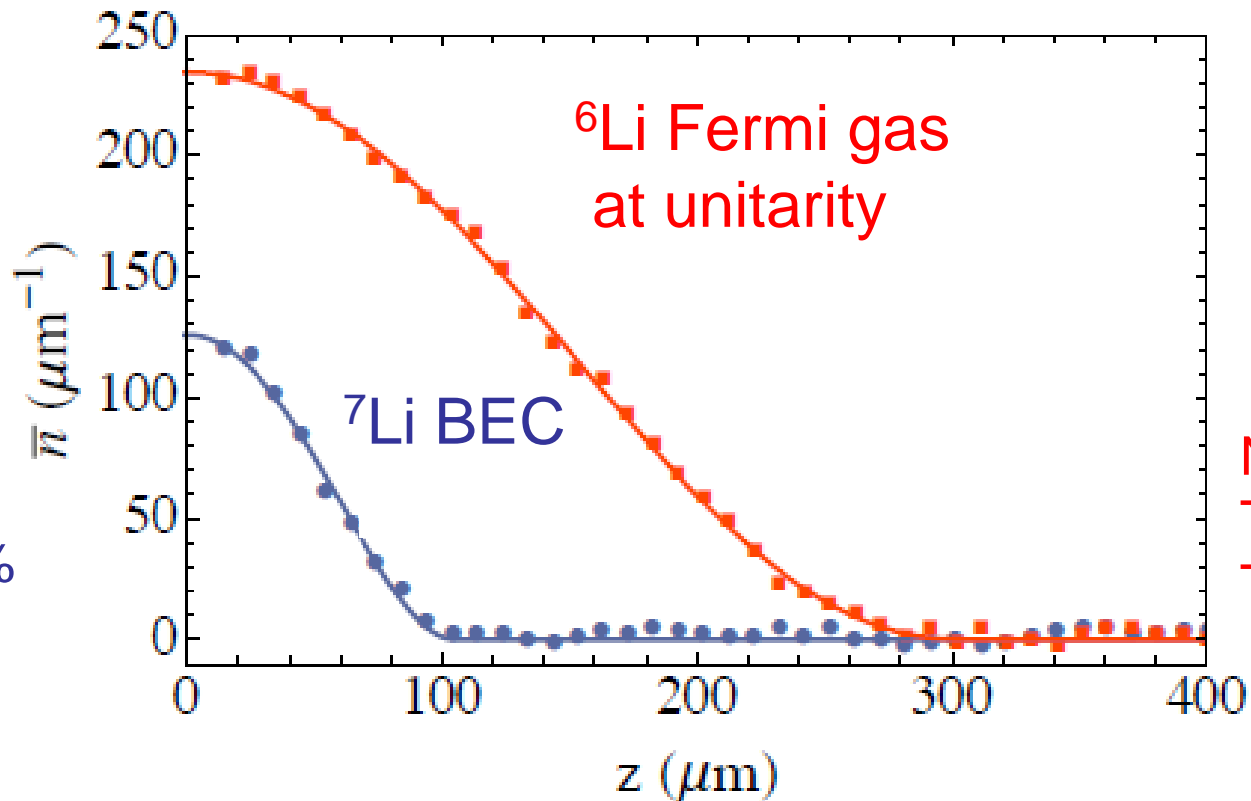
- Low a_{bf} (no interspecies demixing)
- High $|a_f|$ (high fermionic superfluid T_c)
- Positive a_{bb} (stable BEC)

${}^6\text{Li} - {}^7\text{Li}$ mixture in the $|1\rangle_f$, $|2\rangle_f$ and $|2\rangle_b$



$$a_{bf} = 40.8 a_0$$

In situ density profiles



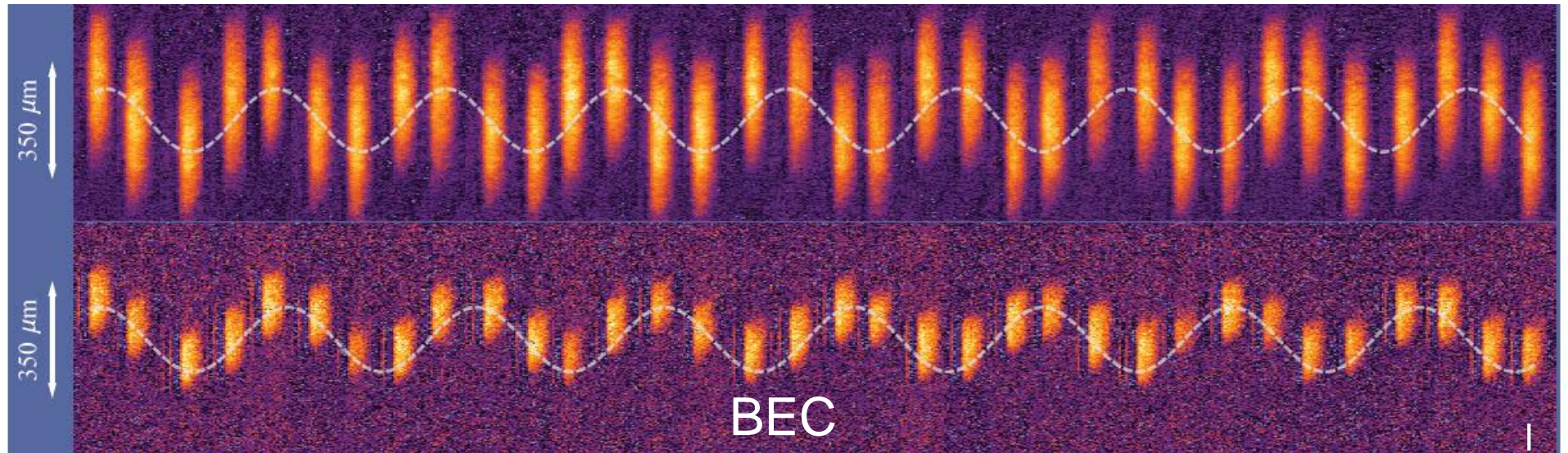
Trap frequencies: $\nu_z = 15.6 \text{ Hz}$
for bosons, $\nu_{\text{rad}} = 440 \text{ Hz}$

Unitary ${}^6\text{Li}$ Fermi gas can cool any species fulfilling the requirements to BEC
See also ${}^6\text{Li}$ - ${}^{41}\text{K}$, USTC, China, PRL '16, and ${}^6\text{Li}$ - ${}^{173}\text{Yb}$, UWash, PRL'17
Cool molecules to quantum regime ?

Long-lived Oscillations of both Superfluids

Fermi Superfluid

time



$$\tilde{\omega}_6 = 2\pi \times 17.06(1) \text{ Hz}$$

$$\omega_6 = 2\pi \times 17.14(3) \text{ Hz}$$

$$\tilde{\omega}_7 = 2\pi \times 15.40(1) \text{ Hz}$$

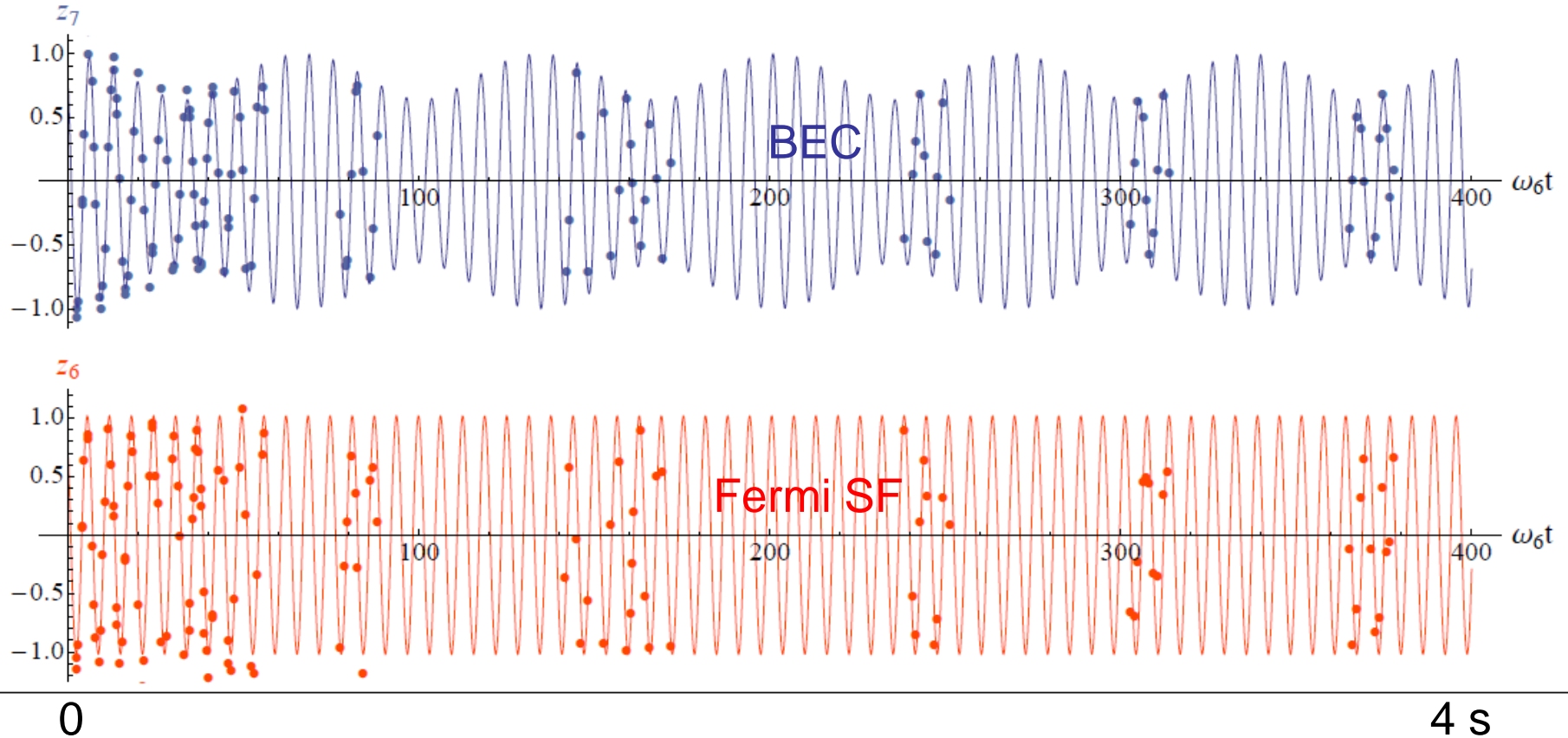
$$\omega_7 = 2\pi \times 15.63(1) \text{ Hz}$$

Coupled Superfluids

Single Superfluid

$$\text{Ratio} = (7/6)^{1/2} = (m_7/m_6)^{1/2}$$

Oscillations of both superfluids



Very small damping !

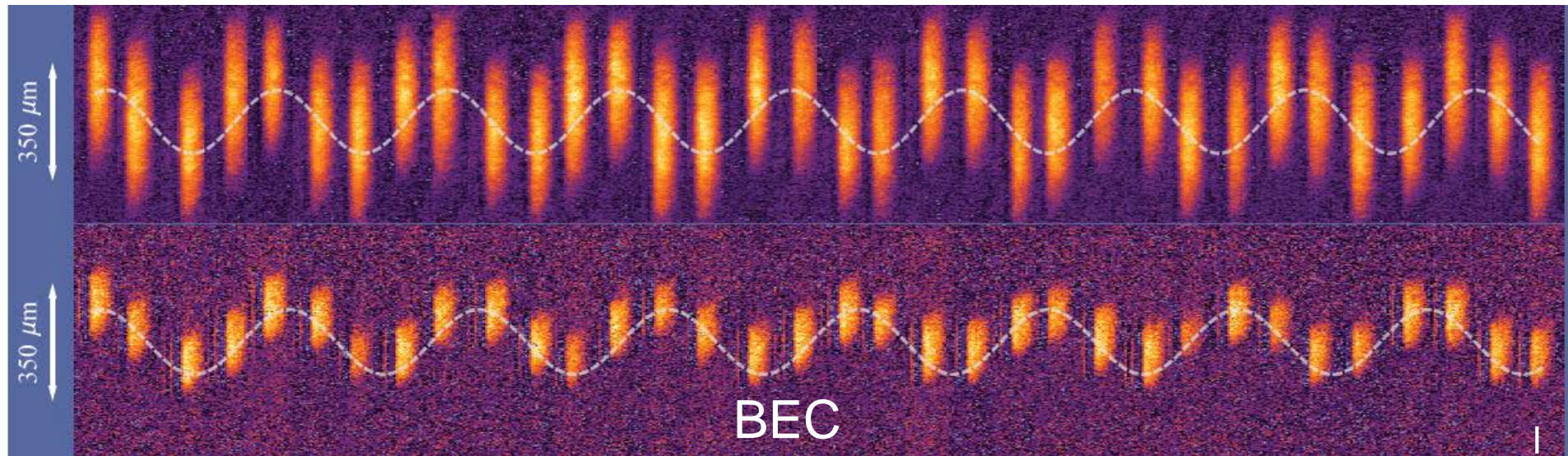
Modulation of the ${}^7\text{Li}$ BEC amplitude by $\sim 30\%$ at $(\tilde{\omega}_6 - \tilde{\omega}_7) / 2\pi$

Coherent energy exchange between the two oscillators

Dual Bose-Fermi superfluids with ^6Li - ^7Li isotopes

Fermi Superfluid

time



400 ms

Question 1: How to understand the oscillation frequencies ?

Question 2: what is the critical velocity for superfluid counterflow ?

Question 3: what is the lifetime of the Bose-Fermi mixture ?

At unitarity, the lifetime is 7 seconds in shallow optical trap

How does it vary with $1/k_f a_f$, with a_{bf} , and with density ?

Mean field model

1.5% down shift in ${}^7\text{Li}$ BEC frequency

BEC osc. amplitude beat at frequency $(\tilde{\omega}_6 - \tilde{\omega}_7) / 2\pi$

Weak interaction regime: $k_F a_{bf} \ll 1$ and $N_7 \ll N_6$

Boson effective potential $V_{eff} = V(r) + g_{bf} n_6(r)$ with $g_{bf} = \frac{2\pi\hbar^2 a_{bf}}{m_{67}}$

$$m_{67} = m_6 m_7 / (m_6 + m_7)$$

LDA $n_6(r) = n_6^0(\mu_6^0 - V(r))$

Where $n_6(\mu)$ is the Eq. of State of the stationary Fermi gas.

For the small BEC: $V(r) \ll \mu_6^0$

Expand $n_6(r) \approx n_6^0(\mu_6^0) - V(r) \frac{dn_6^0}{d\mu_6} + \dots$

Boson effective potential and link with Equation of State

Thomas Fermi radius of BEC \ll TF radius of Fermi Superfluid:

$$V_{eff} = g_{bf} n_6(0) + V(r) \left[1 - g_{bf} \left(\frac{dn_6^{(0)}}{d\mu_6} \right)_0 \right]$$

The potential remains harmonic with rescaled frequency

$$\tilde{\omega}_7 = \omega_7 \sqrt{1 - g_{bf} \left(\frac{dn^{(0)}}{d\mu_6} \right)_0}$$

A new means to access properties of the EoS !

The equation of state $n(\mu)$ at low T is known in the BEC-BCS crossover
N. Navon et al., Science, 2010

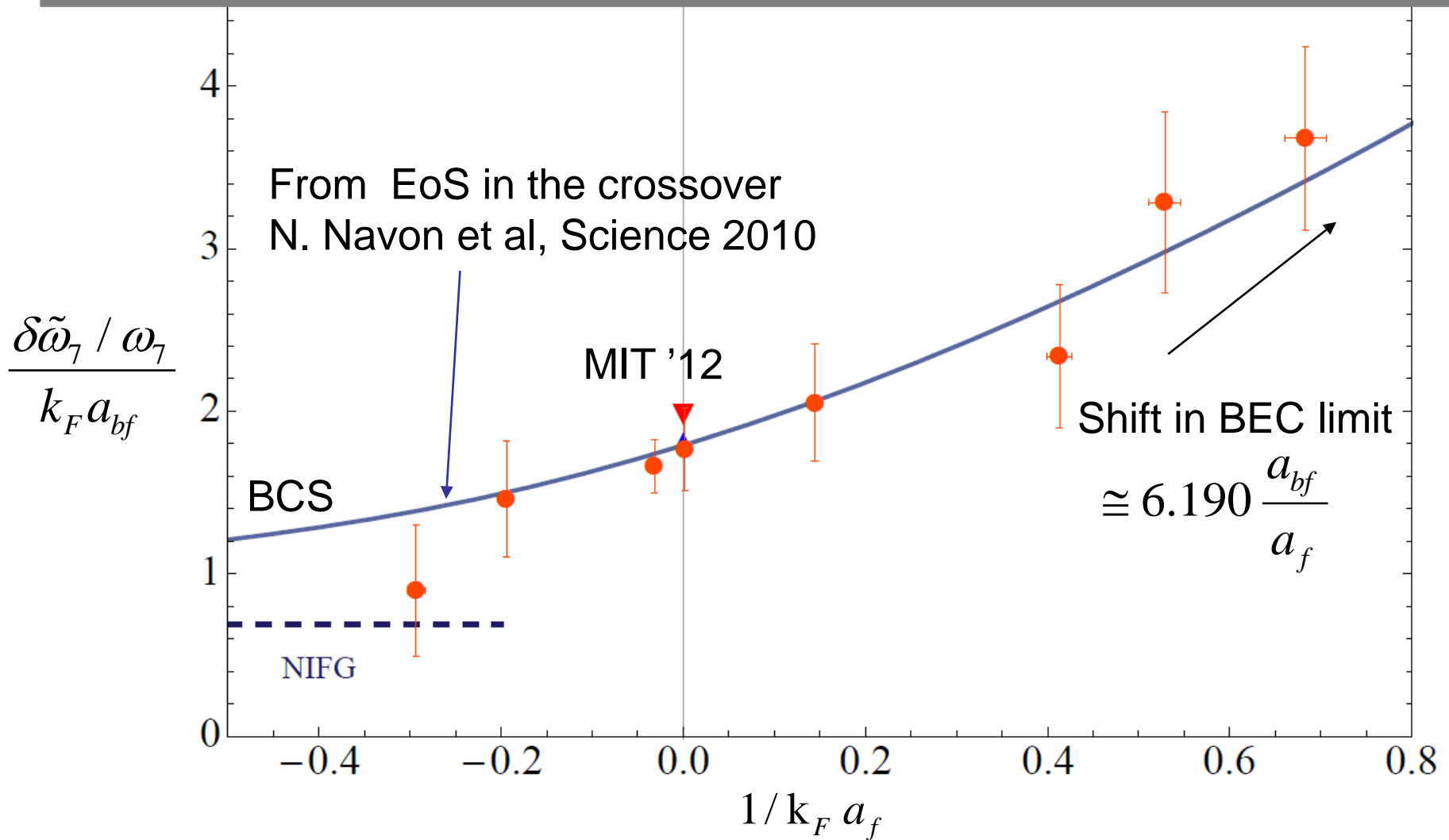
Example: at unitarity, $1/a=0$

From Thomas Fermi radius of ${}^6\text{Li}$ superfluid, we find: $\tilde{\omega}_7 = 2\pi \times 15.43 \text{ Hz}$

very close to the measured value:

$$\tilde{\omega}_7 = 2\pi \times 15.40(1) \text{ Hz}$$

Equation of State and Bose-Fermi Coupling in BEC-BCS crossover



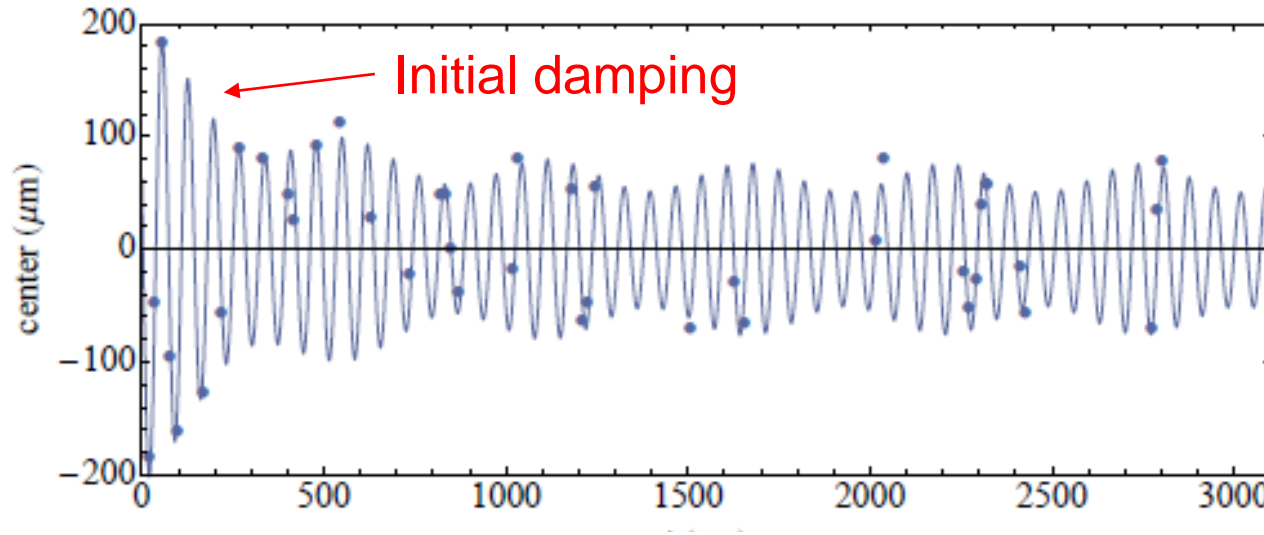
What is the critical velocity for superfluid counterflow ?

Increase initial displacement



Increase relative velocity

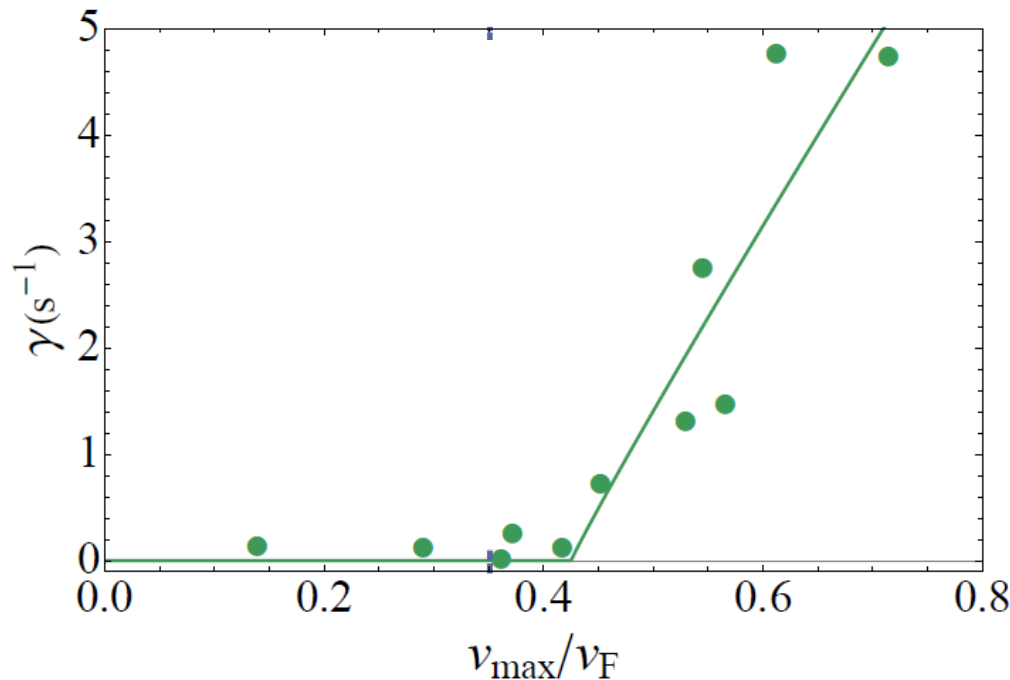
Critical velocity for superfluid counterflow



$$d = d_0 \exp(-\gamma t) + d'$$

$$\gamma = 3.1 \text{ s}^{-1}$$

Time(ms)



$V_c = 2 \text{ cm/s}$
is quite high !

Landau criterion



Momentum Conservation : $M\mathbf{V} = M\mathbf{V}' + \hbar\mathbf{k}$

Energy Conservation : $MV^2 / 2 = MV'^2 / 2 + \epsilon_k$

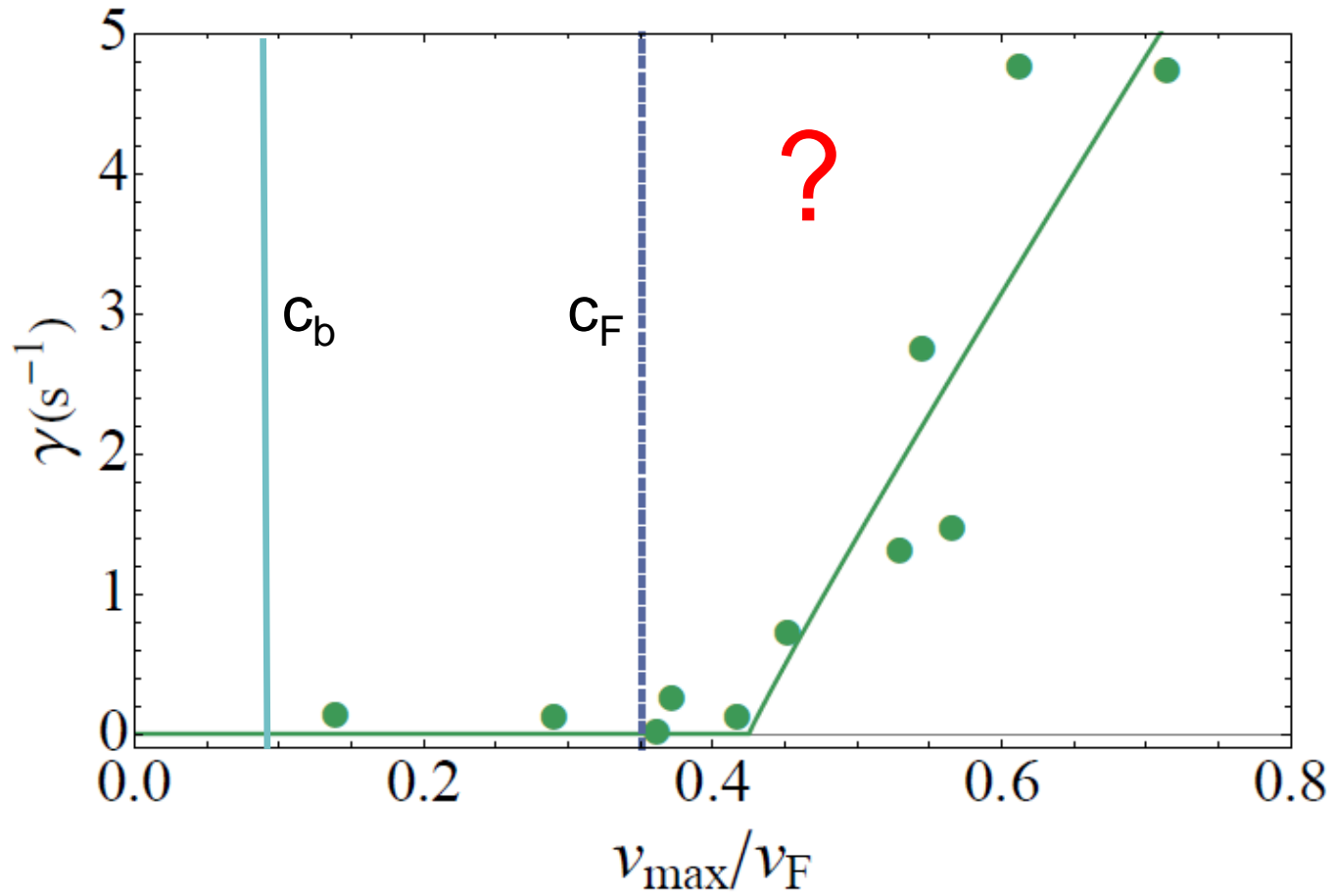
$$\hbar kV \geq \hbar\mathbf{k} \cdot \mathbf{V} = \epsilon_k + \hbar^2 k^2 / 2M \geq \epsilon_k$$

Motion of impurity is damped by the creation of elementary excitations if:

$$V \geq V_c = \min_k \left(\frac{\epsilon_k}{\hbar k} \right)$$

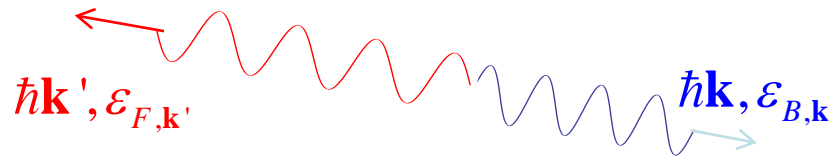
For a linear excitation spectrum $\epsilon_k = \hbar kc$, $V_c = c$, the sound velocity

Critical velocities



Landau criterion for a Bose-Fermi mixture @ T=0

Y. Castin, I. Ferrier-Barbut and C. Salomon
Comptes-Rendus Acad. Sciences, Paris, **16**, 241 (2015)



1 Excitation in the bosonic superfluid

$$E_{B,k} = \varepsilon_{B,k} + \hbar \mathbf{k} \cdot \mathbf{V}_B$$

1 Excitation in the fermionic superfluid

$$E_{F,k'} = \varepsilon_{F,k'} + \hbar \mathbf{k}' \cdot \mathbf{V}_F$$

Energy-momentum conservation:

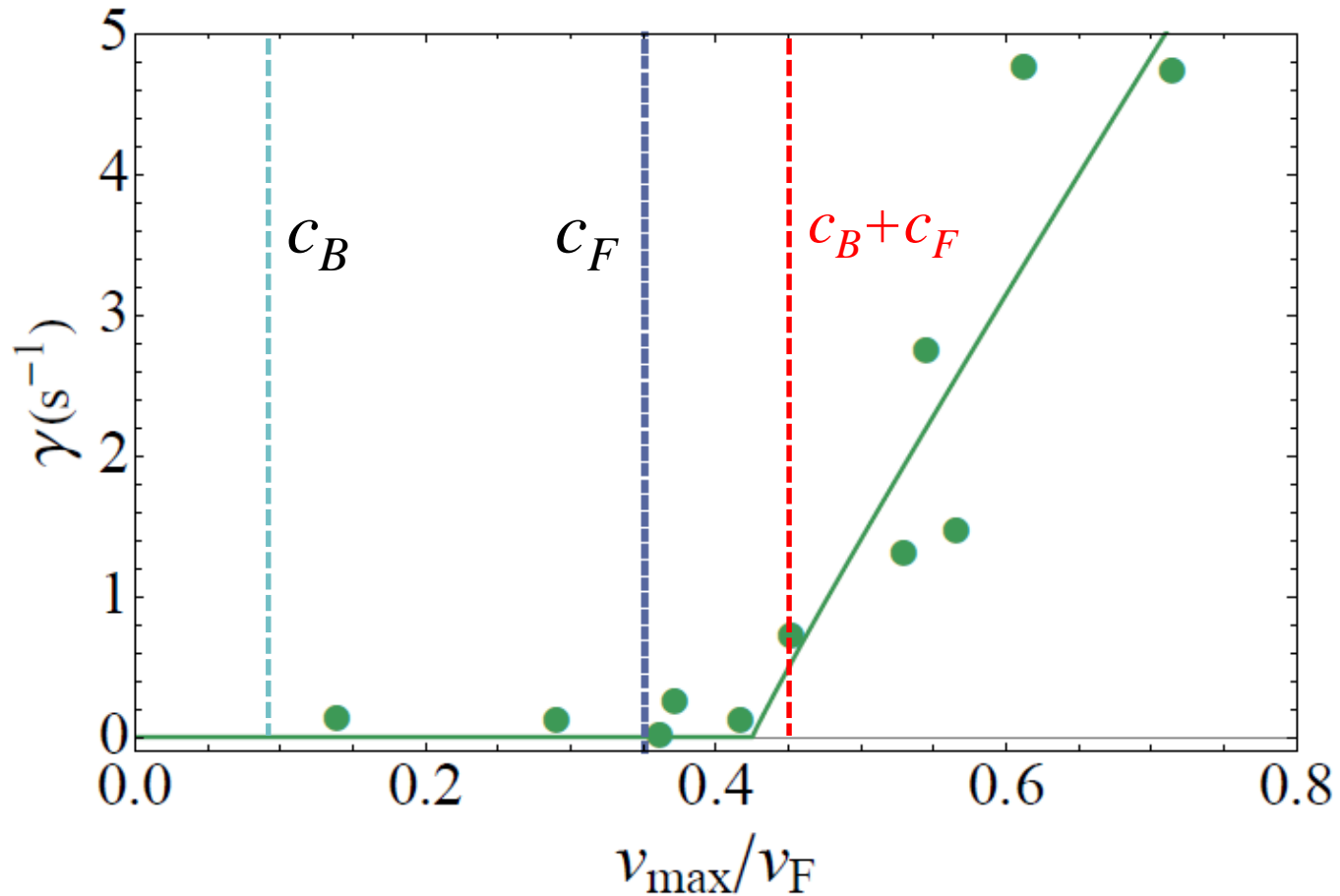
$$E_{B,k} + E_{F,k'} = 0 \quad \mathbf{k} + \mathbf{k}' = 0$$

$$|\mathbf{V}_B - \mathbf{V}_F| \geq \min_k \left(\frac{\varepsilon_{B,k} + \varepsilon_{F,-k}}{\hbar k} \right)$$

Sound Modes: $V_c = c_B + c_F$

See also Abbad et al. EPJD 69, 126 (2015), F. Chevy, PRA **91**, 063606 (2015),
W. Zheng and H. Zhai, Phys. Rev. Lett. 113, 265304 (2014)

Counterflow critical velocity



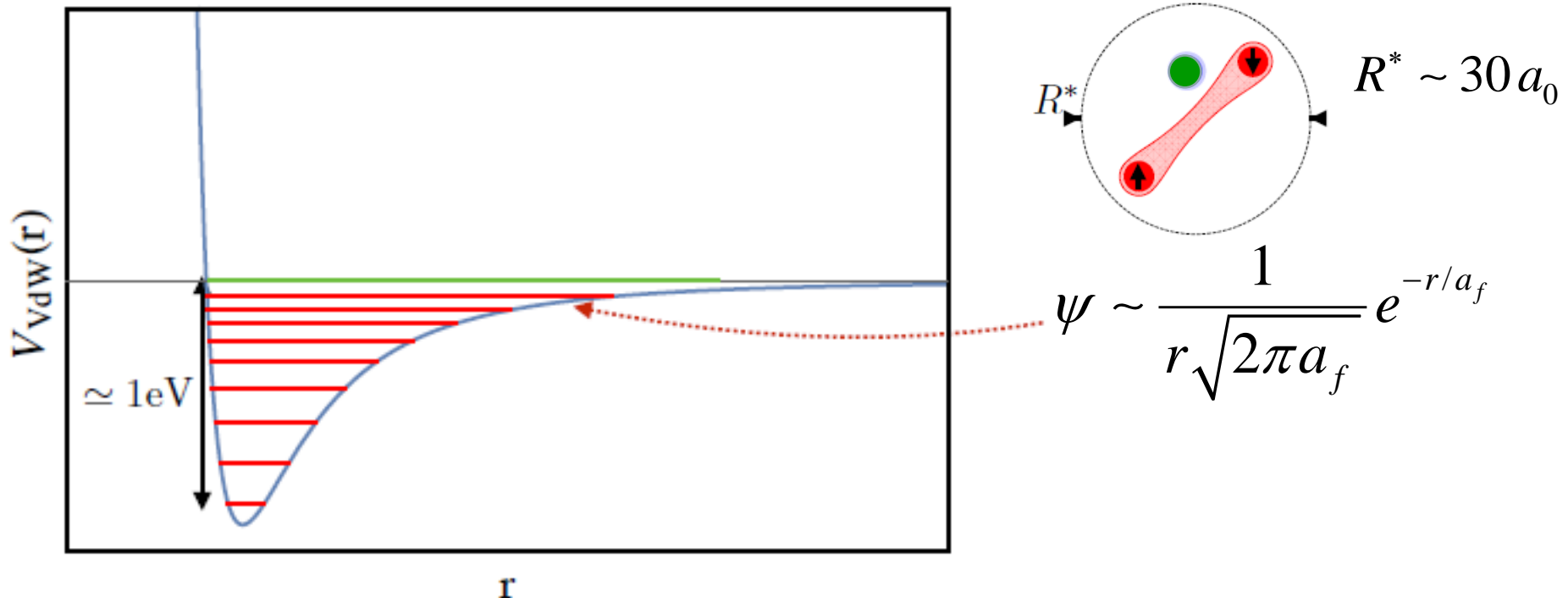
Question 3:
What is the lifetime of the
Bose-Fermi mixture ?

Three-body recombination as a probe of
quantum correlations in a strongly interacting system

Three-body recombination in Bose-Fermi mixture

As a_{bf} is small, bosons act as a weakly coupled impurity i immersed in a Fermi gas with large a_f

Three-body recombination: i, \downarrow, \uparrow



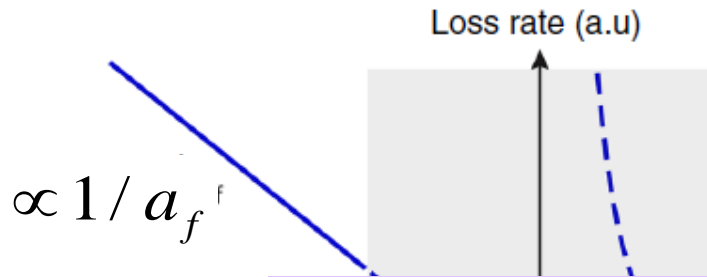
Decay to a deeply bound molecular state

Binding energy transferred to kinetic energy of collision partners

Atom and molecule leave the trap

A weakly coupled impurity in a resonant Fermi gas

D'Incao and Esry, PRL 2008
 Zirbel et al., PRL 2008
 Spiegelhalder et al., PRL 2009
 Khramov et al., PRA 2012



Unitary regime

Assuming a saturation

$$a \simeq n_f^{-1/3}$$

We expect :

$$\Gamma_{if} \propto n_f^{4/3}$$

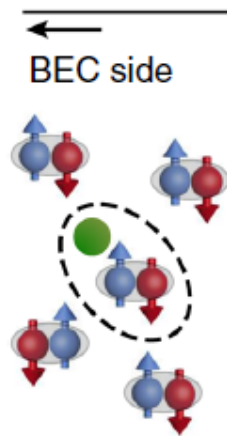
Unitarity: ?

BEC Side

« Two » body
 Dimer-impurity losses

$$\dot{n}_i = -L_{di} n_d n_i$$

$$L_{di} \propto 1/a_f$$



$$\propto a_f^2$$

BCS Side

Three body
 losses

$$\dot{n}_i = -L_{ffi} n_i n_f^2$$

$$L_{ffi} \propto a_f^2$$



A weakly coupled impurity in a resonant Fermi gas

Kagan, Svistunov, Shlyapnikov, JETP, 1985

$$P(R < R^*) = \int_{R < R^*} d^3\mathbf{r}_1 d^3\mathbf{r}_2 d^3\mathbf{r}_3 \langle \hat{\Psi}_1^\dagger(\mathbf{r}_1) \hat{\Psi}_2^\dagger(\mathbf{r}_2) \hat{\Psi}_i^\dagger(\mathbf{r}_3) \hat{\Psi}_i(\mathbf{r}_3) \hat{\Psi}_2(\mathbf{r}_2) \hat{\Psi}_1(\mathbf{r}_1) \rangle$$

Weak coupling between the impurity and the resonant fermions



$$P(R < R^*) = \int_{R < R^*} d^3\mathbf{r}_1 d^3\mathbf{r}_2 d^3\mathbf{r}_3 \underbrace{\langle \hat{\Psi}_1^\dagger(\mathbf{r}_1) \hat{\Psi}_2^\dagger(\mathbf{r}_2) \hat{\Psi}_2(\mathbf{r}_2) \hat{\Psi}_1(\mathbf{r}_1) \rangle}_{g_{\uparrow\downarrow}(r_2, r_1) \times n_i} \langle \hat{\Psi}_i^\dagger(\mathbf{r}_3) \hat{\Psi}_i(\mathbf{r}_3) \rangle$$

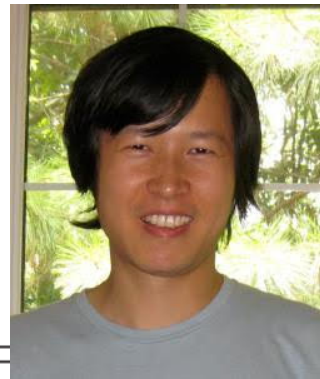
With :

$$g_{\uparrow\downarrow}(r_2, r_1) \underset{|r_2 - r_1| \rightarrow 0}{\sim} \frac{1}{\Omega} \frac{C_2}{4\pi^2 |r_2 - r_1|^2}$$

S. Tan, 2008

Therefore the impurity decay rate Γ_{if} should be proportional to Tan's two-body contact C_2

Tan's Contact



Tail of the momentum distribution at large k

$$k^4 n_\sigma(k) \rightarrow C_2 \quad \text{when } k \rightarrow \infty$$

JILA: Stewart et al., Jin's group, PRL, 2010

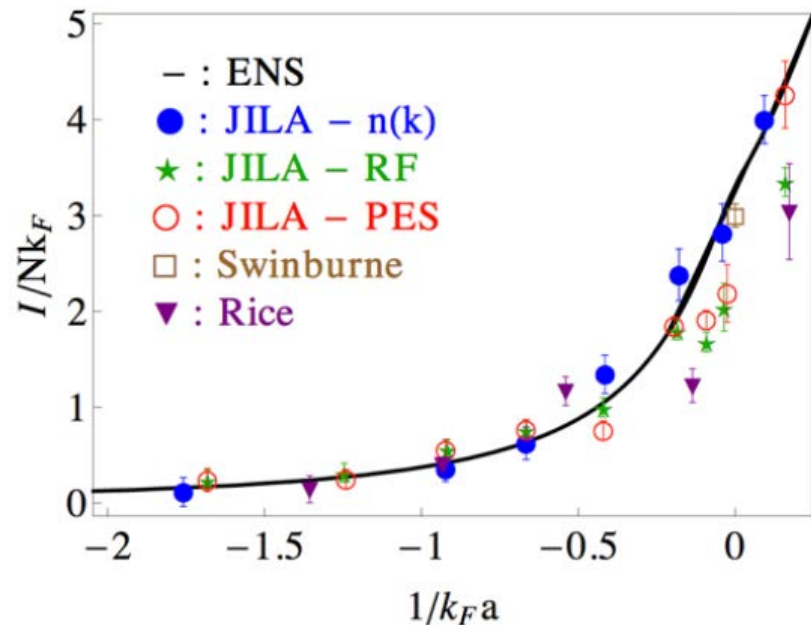
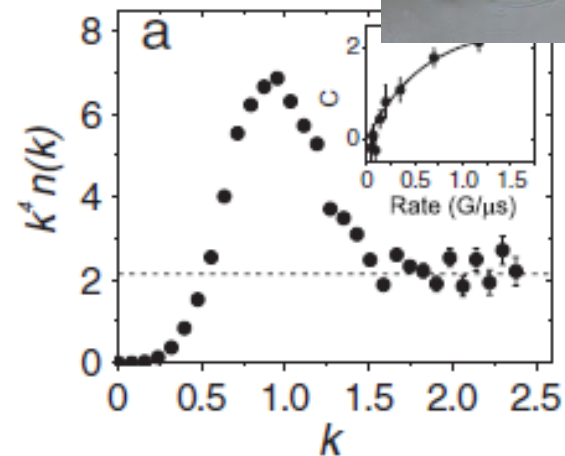
Adiabatic energy relation

$$C_2 = -\frac{4\pi m_f}{\hbar^2} \frac{\partial E}{\partial(1/a)}$$

at constant entropy

From equation of state measurements:

ENS, Navon et al., Science, 2010



Bose/Fermi decay and Tan's Contact

$$\dot{n}_b = -\gamma C_2 n_b = -\Gamma_{bf} n_b$$

$\gamma \propto a_{bf}^2$ is the only parameter that contains short range physics easily measured at high temperature on BEC side

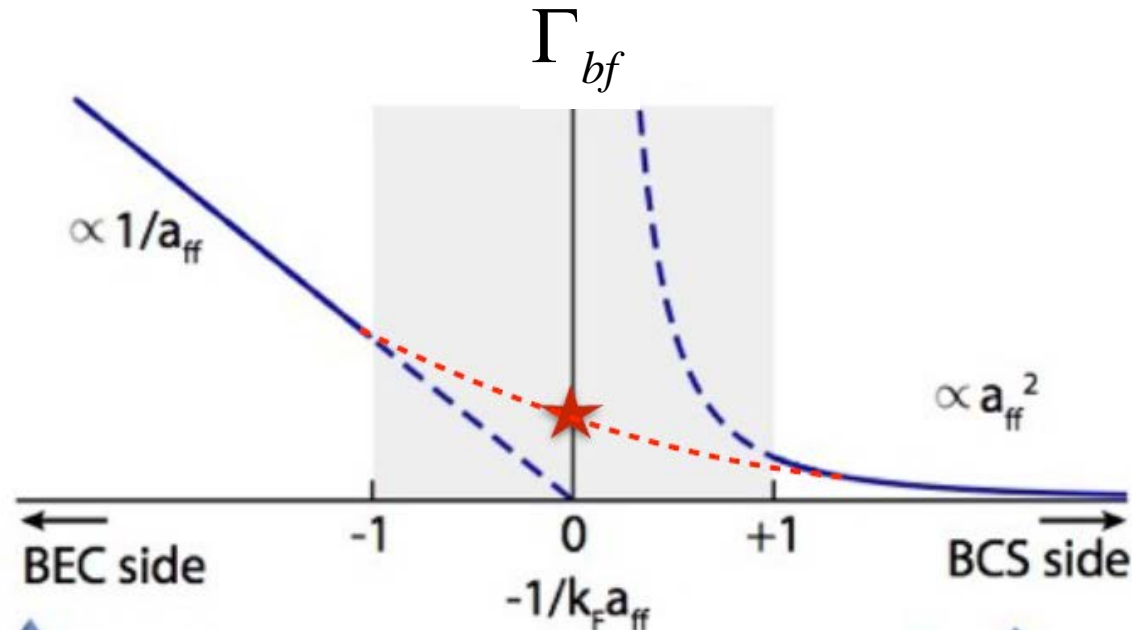
	BEC	Unitary	BCS
(\dot{n}_b/n_b)	$\propto (n_m/a_{ff})$ [20]	$\propto n_f^{4/3}$	$\propto a_{ff}^2 n_f^2$ [20]
C_2	$8\pi(n_m/a_{ff})$	$(2\zeta/5\pi)k_F^4$	$4\pi^2 a_{ff}^2 n_f^2$

$$\zeta = 0.87(3)$$

C. Vale, Swinburne

Bose/Fermi decay in strongly interacting regime

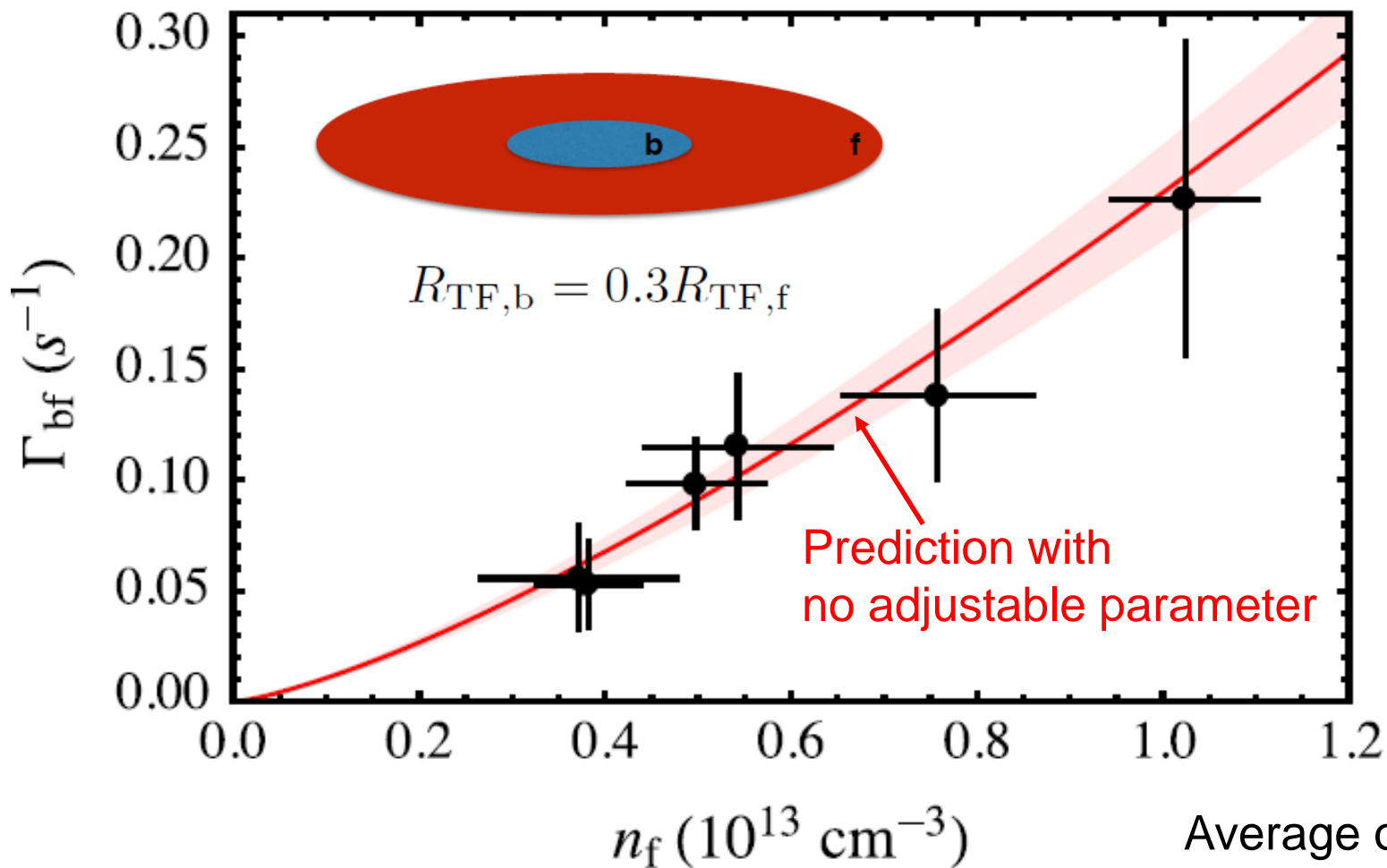
$$\dot{n}_b = -\gamma C_2 n_b = -\Gamma_{bf} n_b$$



BEC + Fermi Superfluid

$$\Gamma_{bf} \propto n_f^{4/3}$$

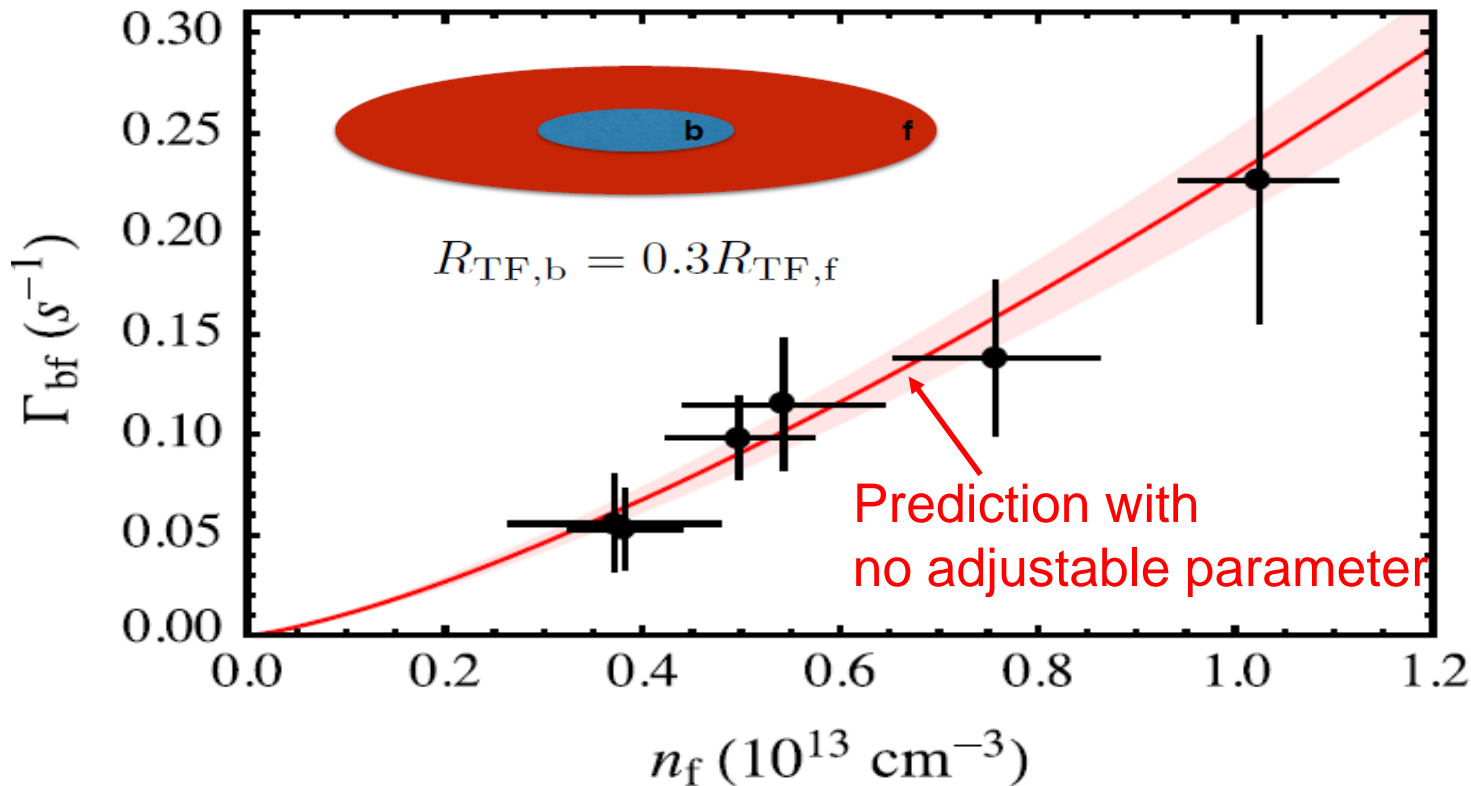
Probing the local unitary Contact



$$\Gamma_{bf} = \gamma C_2 = \frac{2\zeta}{5\pi} \left(3\pi^2 n_f^{4/3} \right) \times 0.9$$

Average over BEC
TF profile

Probing the local unitary Contact



Power law fit: $A n^p$ gives $p = 1.36$ (15) close to $4/3$

Fit: $A n^{4/3}$ gives A and local contact $C_2(0)$

Impurity decay is a local probe of quantum correlations in a many-body system

Summary

- Dual Bose-Fermi superfluids have intriguing novel properties
- Lifetime of Bose-Fermi mixture is governed by Tan's contact
- Theory applies to Yb/⁶Li, K/⁶Li, Rb/⁴⁰K,.....assumes small a_{bf}
- What happens when a_{bf} increases ? Efimov effect
- Measure three-body contact of Fermi gas
- Two-body and three-body contact in unitary Bose gas

R. Fletcher et al., *Science* 2017, Cambridge
Link with lifetime measured at JILA

C. E. Klauss et al., *ArXiv* 1704.01206

