Detection of bulk topological features in real time

Pietro Massignan
Main collaborators

- **Theory (Barcelona)**
  - Alexandre Dauphin
  - Maria Maffei

- **Experiment (Naples)**
  - Filippo Cardano
  - Lorenzo Marrucci
  - Maciej Lewenstein

- **ICFO**
  - Institut de Ciències Fotòniques
Outline

• Topology in condensed matter systems

• One-dimensional chiral models
  - static (SSH)
  - periodically-driven
    (photonic quantum walk)
Hall effect

• Classical Hall effect (1879): when current flows in a 2D material, in presence of an out-of-plane B field, there appears a transverse (Hall) current

• Quantum Hall effect (1980): at low temperatures and high-B, the Hall current is quantized!

• Laughlin (1982): robustness due to topology

• TKNN (1982): Kubo formula links conductivity to the Chern number, a topological invariant defined on the occupied bands

\[ R_H \propto \frac{\hbar}{e^2} \]

\[ \propto \frac{\hbar}{2e^2} \]

\[ \propto \frac{\hbar}{4e^2} \]

Thouless, Kohmoto, Nightingale & den Nijs
Topological insulators

- Insulators in the bulk, but have robust current-carrying edge states
- Protected by the non-trivial topology of the bulk bands against local perturbations, like disorder and interactions
- Enormous progresses in the last 10 years (QSH, 3D TIs., 4D QH, …)
- Characterization non-interacting TIs in terms of discrete symmetries
  T: time-reversal
  C: charge-conjugation
  S: chiral
- Beyond the periodic table: Mott / Anderson / crystalline / Floquet TIs, …

Chiu, Teo, Schnyder & Ryu, Rev. Mod. Phys. (2016)
1D chiral systems

polyacetylene
[Nobel prize in chemistry 2000]

ultracold atoms in superlattices
[M. Atala et al., Nat. Phys. 2013]

cavity polaritons
[St. Jean et al., Nat. Phot. 2017]

optical waveguides
[Zeuner et al., PRL 2015]

ultracold atoms in k-space lattices
[Meier et al., Nat. Comm. 2016]

SC qubits in mw-cavities
[Flurin et al., PRX 2017]
SSH model

- Spinless fermions with staggered tunnelings:

  ![Diagram of SSH model]

  - $\exists$ two sublattices
  - $\exists$ a “canonical” basis where $H$ is purely off-diag: $H = \begin{pmatrix} 0 & h^\dagger \\ h & 0 \end{pmatrix}$

- Chiral symmetry: $\Gamma H \Gamma = -H$ ($\Gamma$: unitary, Hermitian, local)

- In mom. space the Hamiltonian is 2*2, $H_k = E_k \mathbf{n}_k \cdot \mathbf{\sigma}$

- In the canonical basis, $\mathbf{n}_k \perp \hat{z}$ $\forall k$ and $\Gamma = \sigma_z$

- Winding:
  - $\mathcal{W} = 0$
  - $\mathcal{W} = 1$

*Su, Schrieffer & Heeger, Phys. Rev. Lett. (1979)*

*Asbóth, Oroszlány, & Pályi, Lecture Notes in Physics (2016)*
The winding $\mathcal{W}$

- $\mathcal{W}$ may be calculated:
  - from $\mathbf{n}$: $\mathcal{W} = \int \frac{dk}{2\pi} (\mathbf{n} \times \partial_k \mathbf{n})_z$
  - from the eigenstates: $\mathcal{W} = \int \frac{dk}{\pi} S,$ $S = i\langle \psi_+ | \partial_k \psi_- \rangle$

- What if the Hamiltonian is not known? Can one measure the winding?

  Yes, and it’s simple!

$H_k = E_k \mathbf{n}_k \cdot \sigma$
Evolution in real time

- Initial condition localized on the $m=0$ cell:

- Mean Chiral Displacement:

$$\mathcal{C}(t) \equiv 2 \left\langle \Gamma m(t) \right\rangle = 2 \int_{-\pi}^{\pi} \frac{dk}{2\pi} \left\langle U^{-t} \sigma_z (i\partial_k) U^{t} \right\rangle_{\psi_0} = 2 \int_{-\pi}^{\pi} \frac{dk}{2\pi} \sin^2(Et) |\mathbf{n} \times \partial_k \mathbf{n}| \xrightarrow{t \to \infty} \mathcal{W}$$

- Easy to measure:

$$\mathcal{C}(t) = 2 \left[ \left\langle m_A(t) \right\rangle - \left\langle m_B(t) \right\rangle \right]$$

- Fast convergence
Resistance to disorder

SSH model in the topological phase $j' = 2j \rightarrow \begin{cases} W = 1 \\ \Delta_{\text{gap}} = 2j \end{cases}$

+ independent disorder of amplitude $\Delta$ on all tunnelings

+ randomly-polarized localized initial condition

+ average over 50 (1000) disorder realizations

\[ \begin{align*}
\Delta &= 0.1 j \\
\Delta &= 1 j \\
\Delta &= 3 j
\end{align*} \]

\[ C \]

the MCD stays locked to the topological invariant as long as $\Delta < \Delta_{\text{gap}}$
Floquet 1D chiral models

photonic quantum walk of twisted photons
Digression: twisted photons

25th anniversary: Allen et al., PRA (1992)

- Collimated monochromatic beam propagating along $\hat{z}$

- Light has linear momentum $\mathbf{p} \propto \mathbf{E}^* \times \mathbf{B}$ ("push")

- But it can also carry also angular momentum

- In the "paraxial approximation", $\hat{J}_z = \hat{S}_z + \hat{L}_z$

- "Spin" AM: $\hat{S}_z = \hbar \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

- Orbital AM: $\hat{L}_z = -i\hbar (\mathbf{r} \times \nabla)_z$

- SAM interaction: circularly polarized light interacts with the particle’s spin

- OAM interaction: light with OAM rotates a particle around the beam axis
Twisting light

- Helical modes have a phase pattern $e^{im\phi}$
- Their OAM is quantized, $\hbar m$

---

**Figure 1.2:** Transverse LG intensity profiles for several radial and azimuthal modes.

**Figure 1.3:** Wave fronts of helical beams with different values of the OAM per photon.

---

Franke-Allen & Radwell

*Optics & Photonics News (2017)*
Q-plates

- Liquid crystals deposited on glass plates along singular patterns cause phase retardation of the beam.

- Q-plates mix OAM and SAM: ("spin-dependent translation")

- An external voltage controls the orientation of the LCs, and therefore the mixing parameter $\delta$.

Discrete-Time Quantum Walk with twisted photons

- Cascade of Q-plates and quarter-wave plates \(\rightarrow\) discretized evolution

- Initial state: \(m=0\) OAM, and a given polarization

\[
\hat{W} = \frac{1}{2} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}
\]

<table>
<thead>
<tr>
<th>Twisted photons</th>
<th>DTQW</th>
</tr>
</thead>
<tbody>
<tr>
<td>OAM ((m))</td>
<td>walker’s position</td>
</tr>
<tr>
<td>polarization ((\mathcal{O}/\overline{\mathcal{O}}))</td>
<td>coin state ((\uparrow/\downarrow))</td>
</tr>
<tr>
<td>Q-plate</td>
<td>conditional displacement</td>
</tr>
<tr>
<td>(\hat{Z})</td>
<td>time</td>
</tr>
</tbody>
</table>

[Cardano et al., Science Advances (2015)]
Discrete-Time Quantum Walk

- Periodic evolution: may be treated via Floquet theory

- One-step evolution operator \( U \) \( \rightarrow \) \( H_{\text{eff}} \equiv i(\log U)/T \)

- In momentum space, \( H_{\text{eff}}(k) = E_k \hat{n}_k \cdot \sigma \)

- The spectrum of \( H_{\text{eff}} \) is \( 2\pi \)-periodic (quasi-energies \( E_k \))

- T+C+S symmetries: BDI class \( \rightarrow \) same invariant as the static SSH model

\[ \text{Cardano, D'Errico, Dauphin, Maffei, ... Marrucci, Lewenstein & PM} \]

\[ \text{Nature Comm. (2017)} \]
Detecting the invariant

- Winding: \( W = \int \frac{dk}{2\pi} (\mathbf{n} \times \partial_k \mathbf{n})_z \)

- Experimental measurement of the MCD after 7 timesteps of the DTQW with twisted photons:

  - Check bulk-boundary correspondence

- Spectrum with edges:

  - Bulk-boundary correspondence violated?
Timeframes

• Different initial $t_0$ lead to different $U$

• Eigenvalues of $H_{\text{eff}}$ don’t depend on $t_0$

• Eigenstates instead do! And so does the winding: $\mathcal{W} = \mathcal{W}_1 \neq \mathcal{W}_2$

• Timeframes invariant under time-reflection ($U_1$ and $U_2$) are special

• # of 0-energy edge states: $C_0 = (\mathcal{W}_1 + \mathcal{W}_2)/2$

• # of $\pi$-energy edge states: $C_\pi = (\mathcal{W}_1 - \mathcal{W}_2)/2$

[Asboth and Obuse, PRB (2013)]
Winding in an alternative timeframe

Measurement of the MCD with protocol $U_2$:

$\sqrt{Q_\delta} = Q_{\delta/2}$
Bulk-boundary correspondence

Theory:

Measurements:
Robustness to noise

- Adding noise to a trivial/non-trivial configuration:

\[ \Delta = \frac{\pi}{10} \]

\[ \Delta = \frac{\pi}{5} \]

(clean \(\phi\)): averages over 10 disorder realizations

(●/●): averages over 10 disorder realizations
Recent developments

• Extension to multi-band models:
  Maffei, Dauphin, …, and PM

• Topological transitions driven by disorder:

• 2D Hofstadter strips (ladders)

Mugel, Dauphin, PM et al.
SciPost Physics 3, 012 (2017)
Conclusions

• The mean chiral displacement captures the winding of 1D chiral systems (both static and periodically driven)

• Detection of MCD is simple, rapid, and robust to disorder and noise

• Topological characterization of Floquet systems by studying different timeframes

• Extending the MCD to other topological classes?

• Interacting systems?

Thank you!