



# Quantum Algorithmic Breakeven: on scaling up with noisy qubits

ICQSIM2017 : International Conference on Quantum Simulation

Paris  
November 15, 2017

Daniel Lidar, USC

To scale up indefinitely we must break even first

## break even

phrase of [break](#)

1. reach a point in a business venture when the profits are equal to the costs.  
"the firm will break even at the operating level this year"

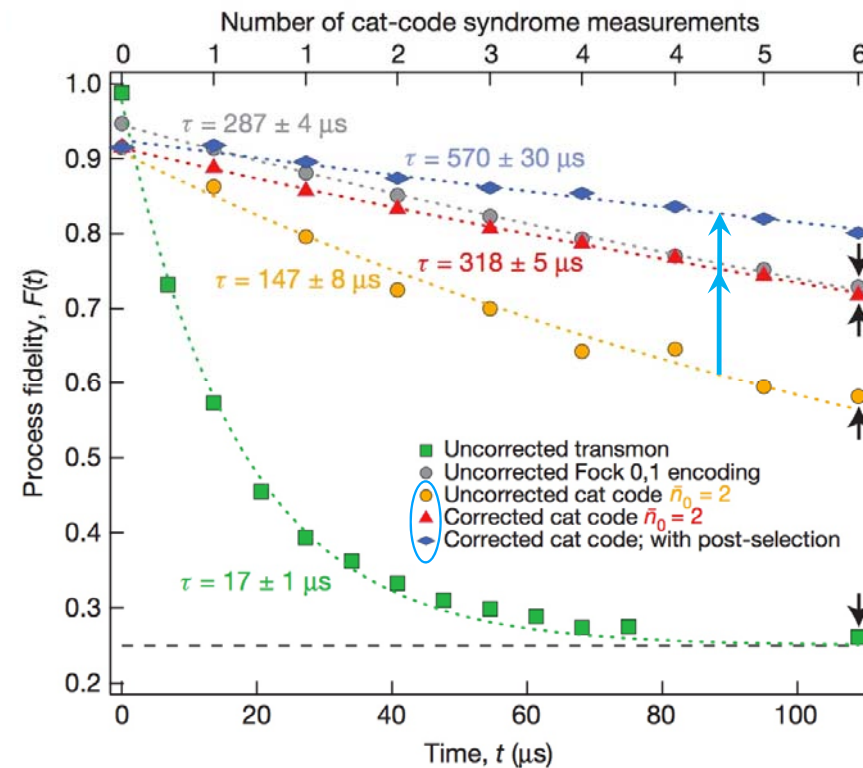
# What is quantum breakeven?

- Achieving the accuracy threshold for fault tolerant quantum computation
  - Performance guarantee is highly model-dependent: threshold =  $x$  means different things for different models (e.g., Markovian vs non-Markovian noise)

- A logical qubit with higher coherence than the constituent physical qubits
- A logical gate with higher fidelity than the constituent physical gates
  - No guarantee this will continue to hold at larger scales (more qubits & gates): unanticipated errors may appear

■ Can we replace with a breakeven notion that

- is model-independent?
- holds at large scales?



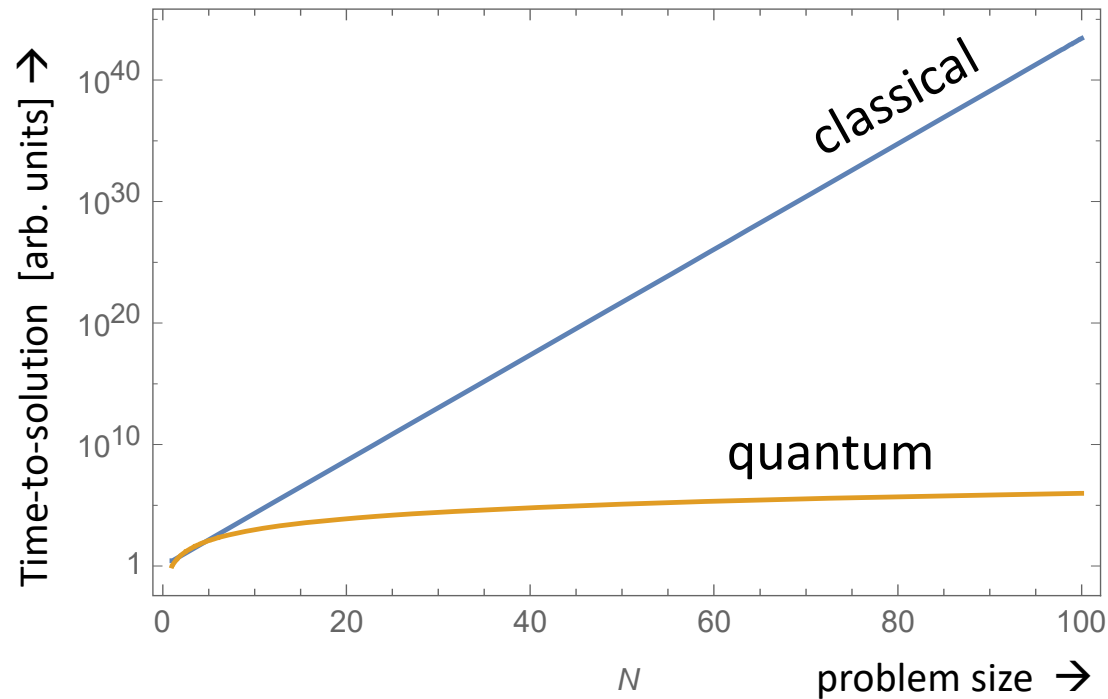
Ofek et al. (Yale group), Nature 536, 441 (2016)

Consider Algorithmic Scaling

# Algorithmic Scaling

Run an algorithm with exponential quantum speedup (e.g., quantum simulation) on quantum hardware

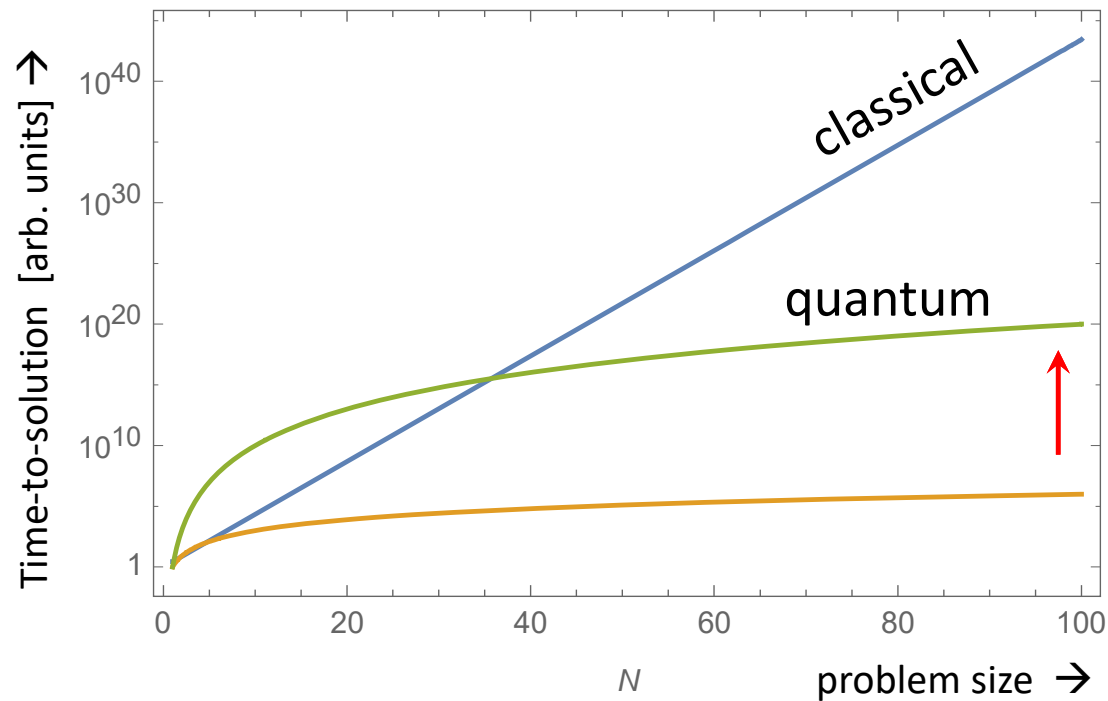
ideal case: no decoherence



quantum scaling  
advantage clear  
for all  $N$

# Algorithmic Scaling

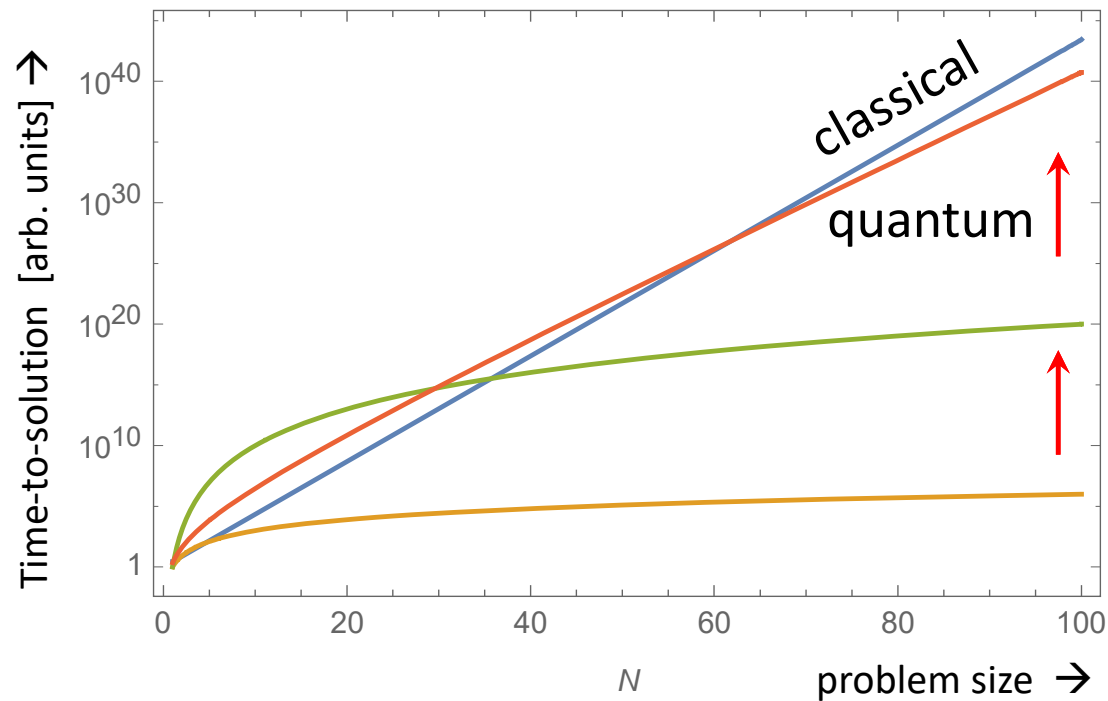
Run an algorithm with exponential quantum speedup (e.g., quantum simulation) on quantum hardware  
with decoherence



quantum scaling  
advantage only  
clear for large  $N$

# Algorithmic Scaling

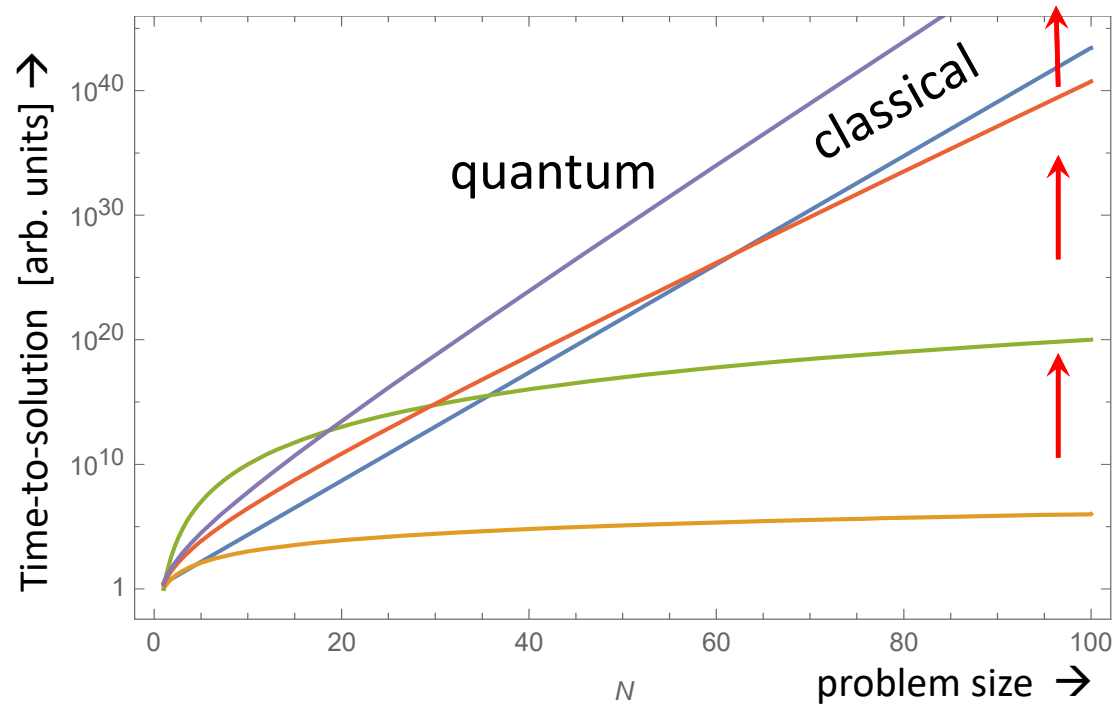
Run an algorithm with exponential quantum speedup (e.g., quantum simulation) on quantum hardware  
with more decoherence



quantum scaling  
advantage only  
clear for even  
larger  $N$

# Algorithmic Scaling

Run an algorithm with exponential quantum speedup (e.g., quantum simulation) on quantum hardware  
with too much decoherence



quantum scaling  
disadvantage

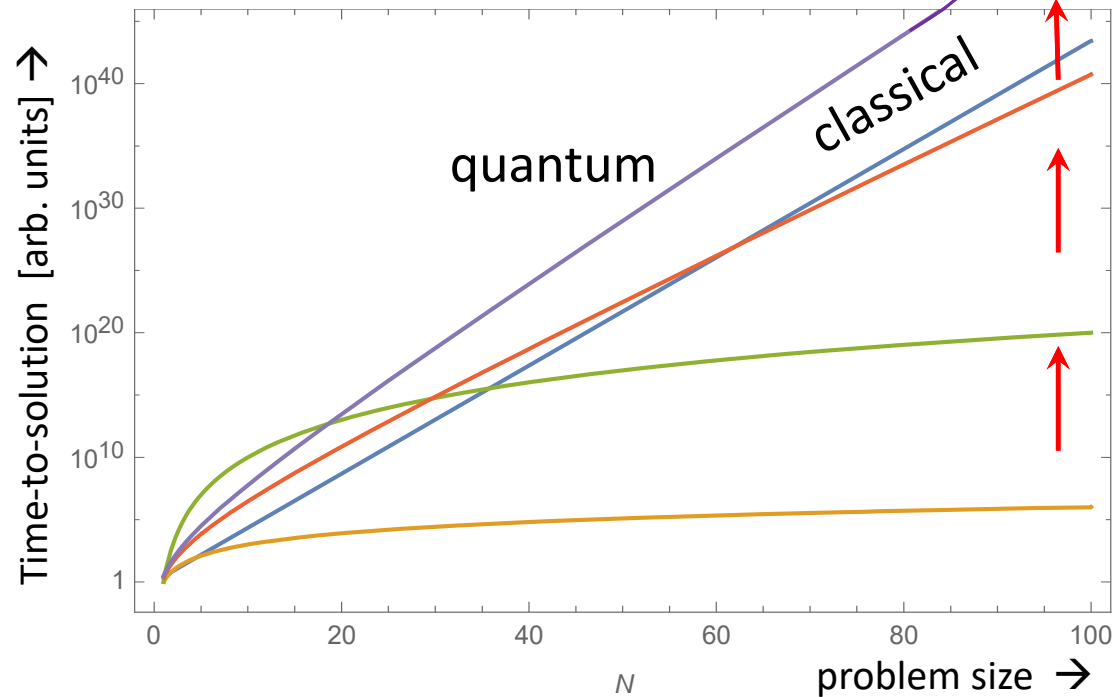


# Algorithmic Scaling

Run an algorithm with exponential quantum speedup (e.g., quantum simulation) on quantum hardware

with too much decoherence

quantum blowup



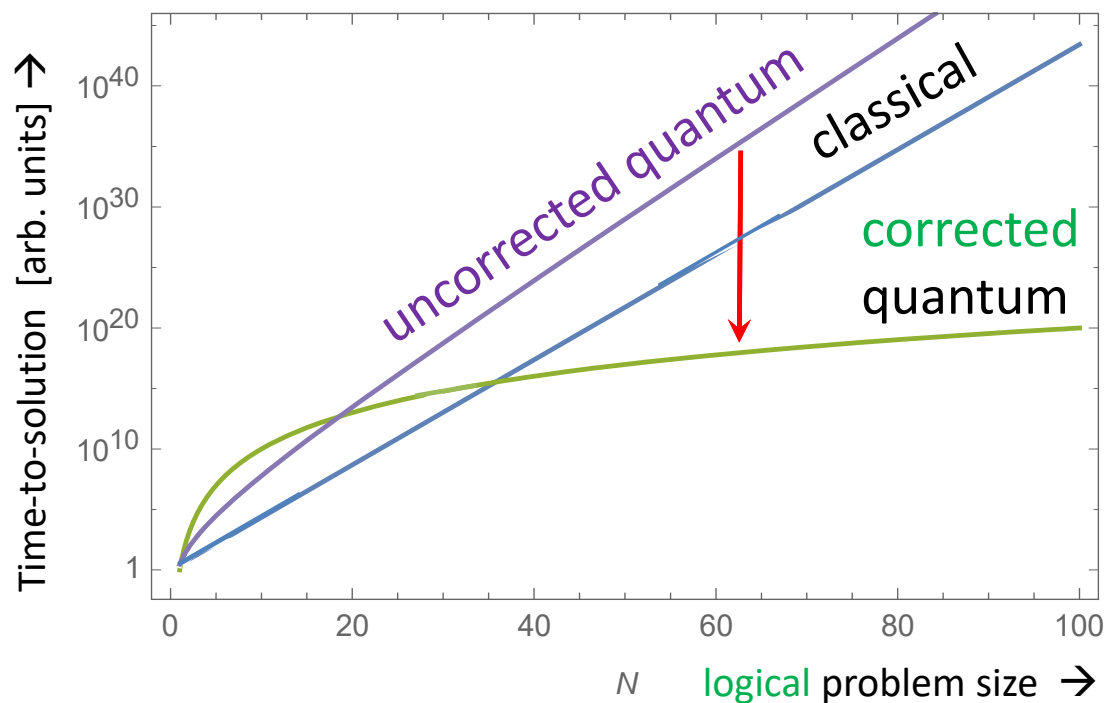
## Enter QEC

Run an algorithm with exponential quantum speedup (e.g., quantum simulation) on quantum hardware  
with decoherence + QEC

# Enter QEC

Run an algorithm with exponential quantum speedup (e.g., quantum simulation) on quantum hardware

with decoherence + QEC



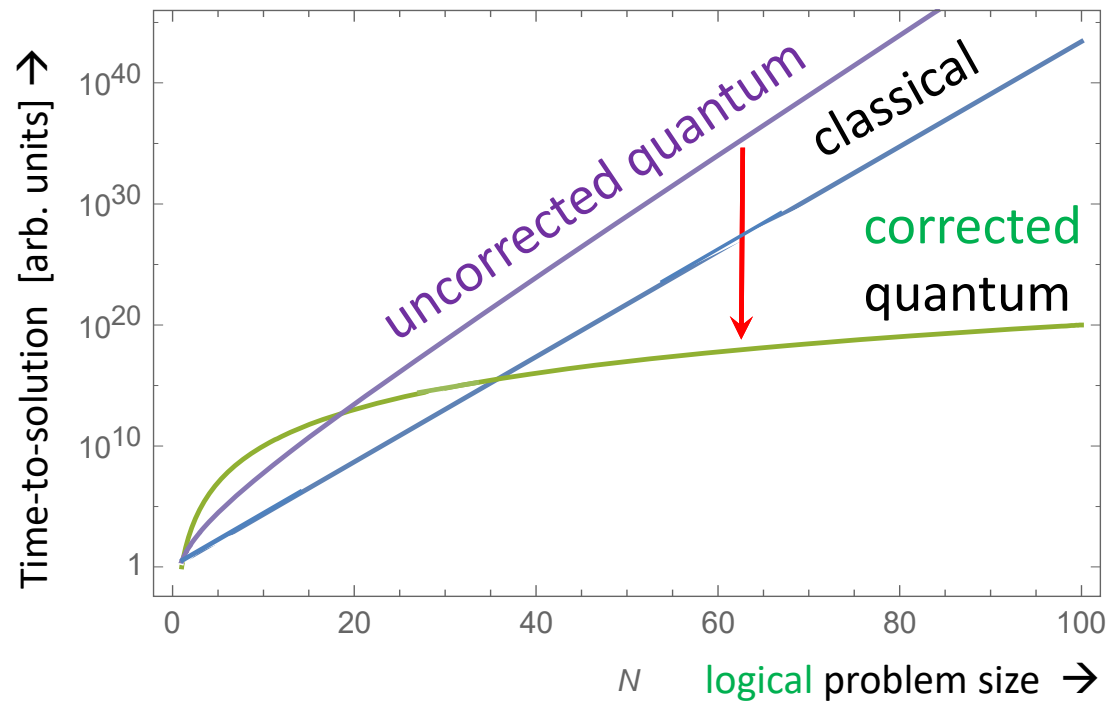
quantum scaling advantage returns

**Algorithmic success with QEC:** corrected quantum scaling is better than classical & uncorrected quantum

# Algorithmic Success with QEC

Run an algorithm with exponential quantum speedup (e.g., quantum simulation) on quantum hardware

with decoherence + QEC



achieving this  
with quantum  
hardware =

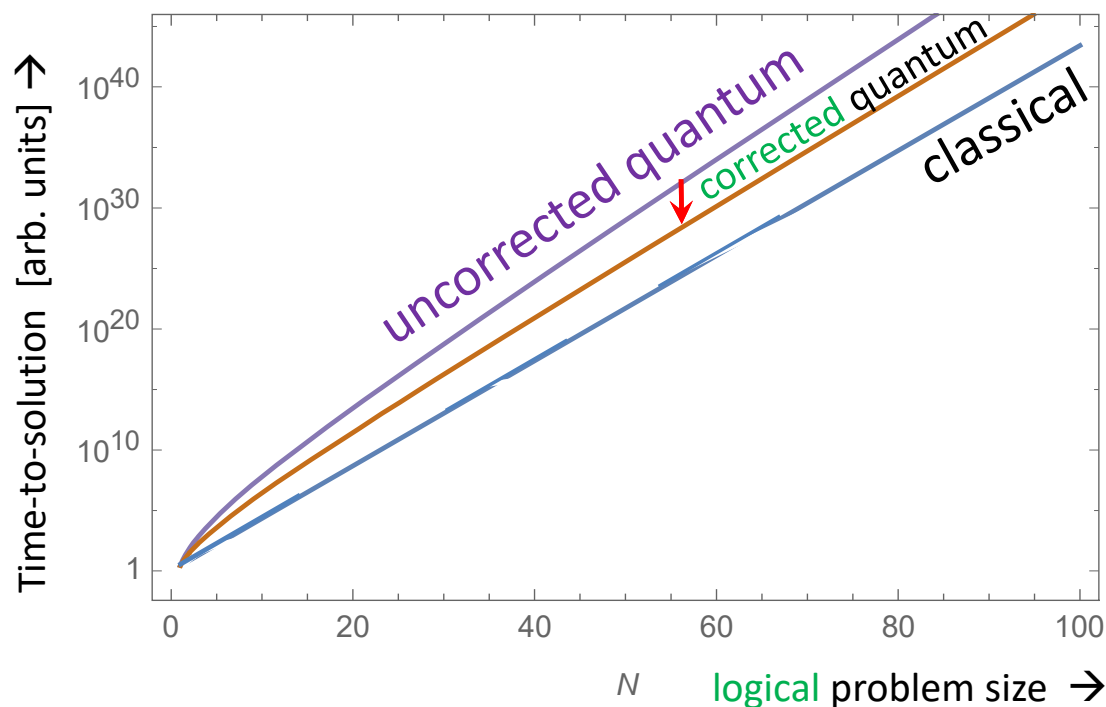


Algorithmic success with QEC: corrected quantum scaling is better than classical & uncorrected quantum

## Algorithmic Breakeven with QEC

Run an algorithm with exponential quantum speedup (e.g., quantum simulation) on quantum hardware

with decoherence + QEC



More modest:

model-independent ✓  
holds at large scales ✓

Can **this** be  
achieved with  
existing quantum  
hardware?

**Algorithmic breakeven with QEC:** corrected quantum scaling is **no worse** than uncorrected quantum,  
but not necessarily better than classical

# Algorithmic breakeven with quantum annealing

K. Pudenz, T. Albash, DL, Nature Comm. 5, 3243 (2014); PRA 91, 042302 (2015)

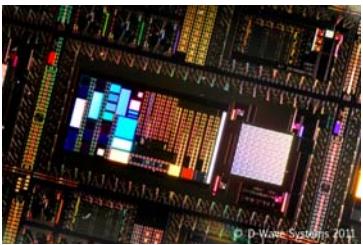
# Algorithmic breakeven with quantum annealing

Problem:  
find ground state energy of

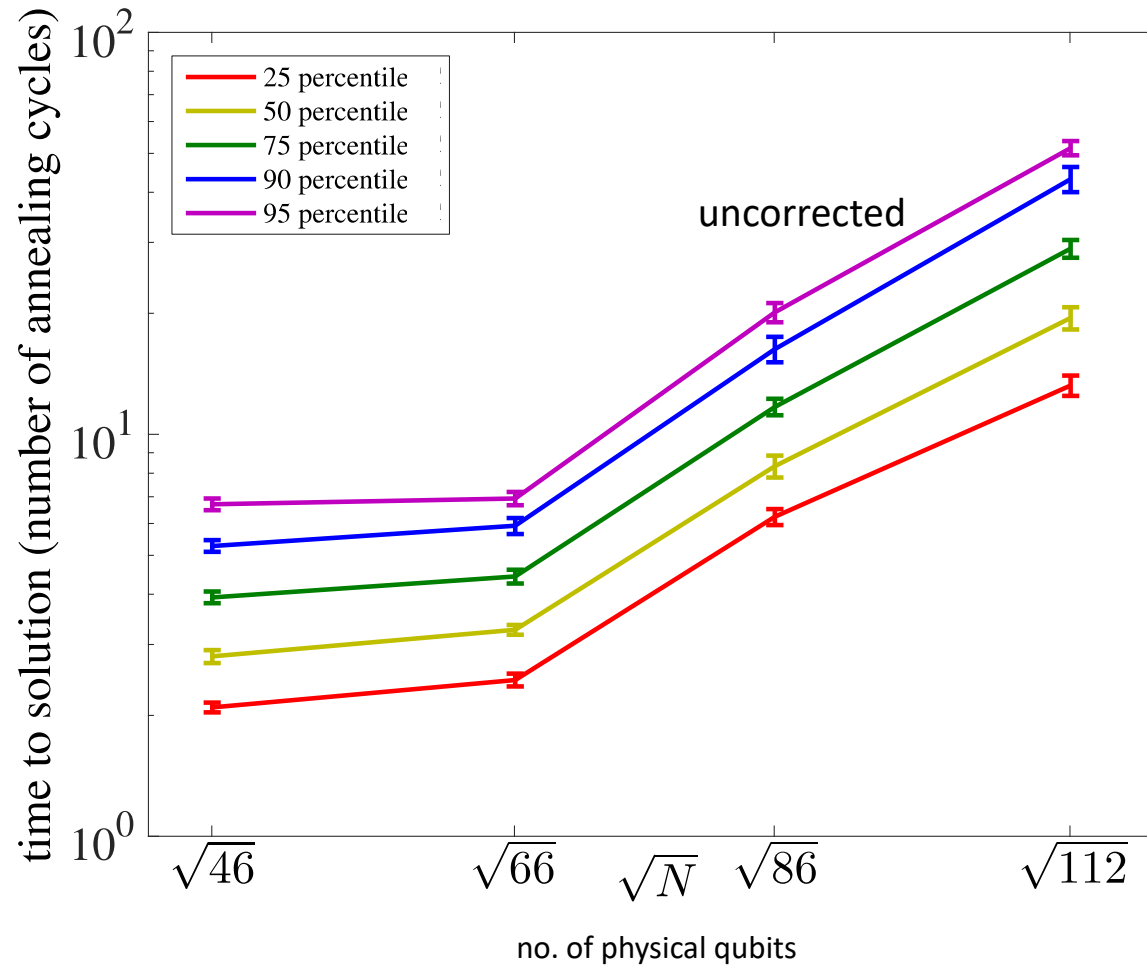
$$H_{\text{Ising}} = \sum J_{ij} \sigma_i^z \sigma_j^z$$

with random

$$J_{ij} \in \left\{ \pm \frac{1}{6}, \dots, \pm \frac{5}{6}, \pm 1 \right\}$$



Run on a D-Wave 2  
(503 qubits)



K. Pudenz, T. Albash, DL, Nature Comm. 5, 3243 (2014); PRA 91, 042302 (2015)

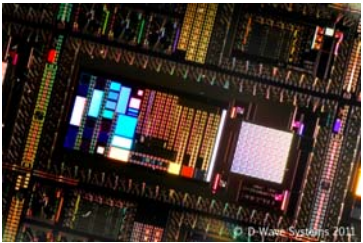
# Algorithmic breakeven with quantum annealing ✓

Problem:  
find ground state energy of

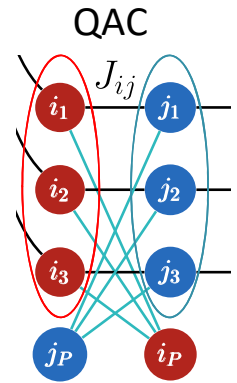
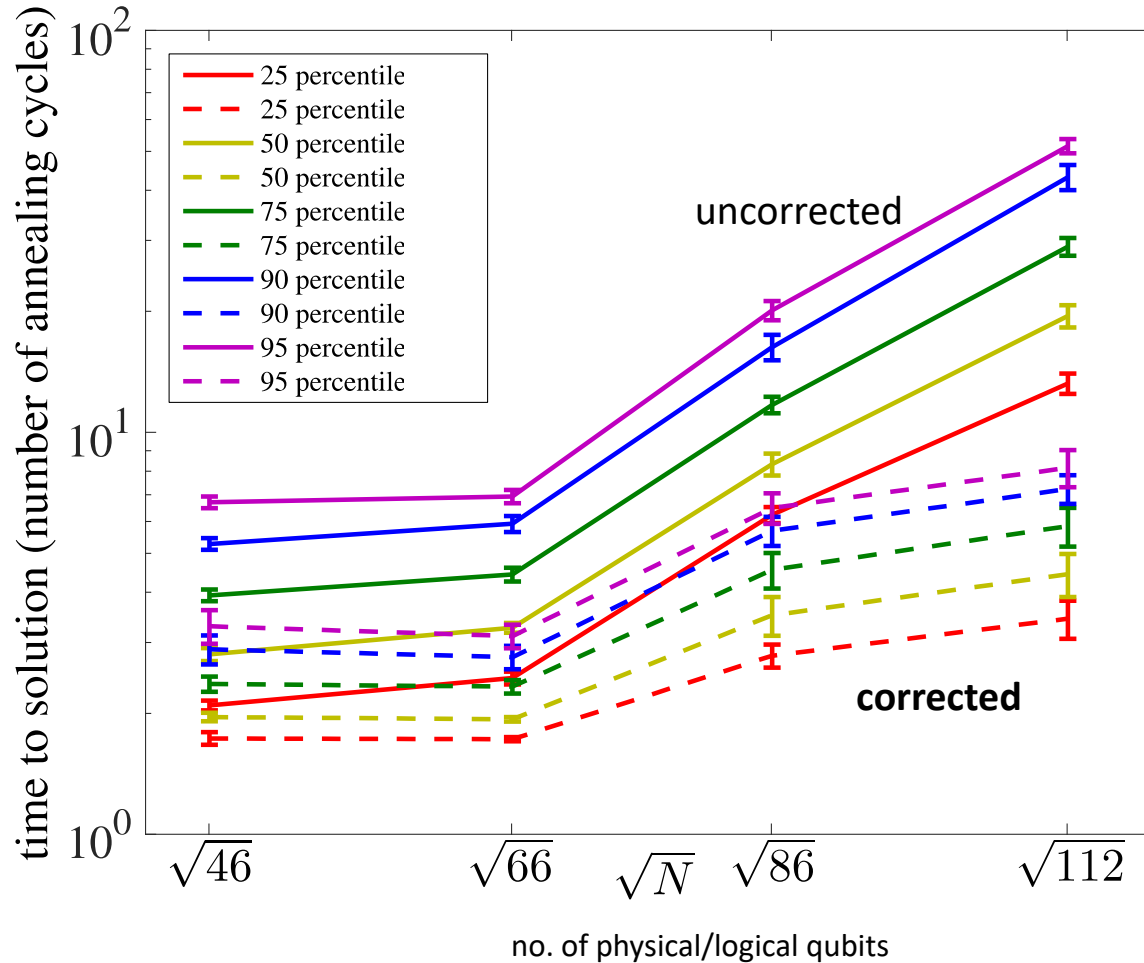
$$H_{\text{Ising}} = \sum J_{ij} \sigma_i^z \sigma_j^z$$

with random

$$J_{ij} \in \left\{ \pm \frac{1}{6}, \dots, \pm \frac{5}{6}, \pm 1 \right\}$$



Run on a D-Wave 2  
(503 qubits)



no claim of q. speedup!

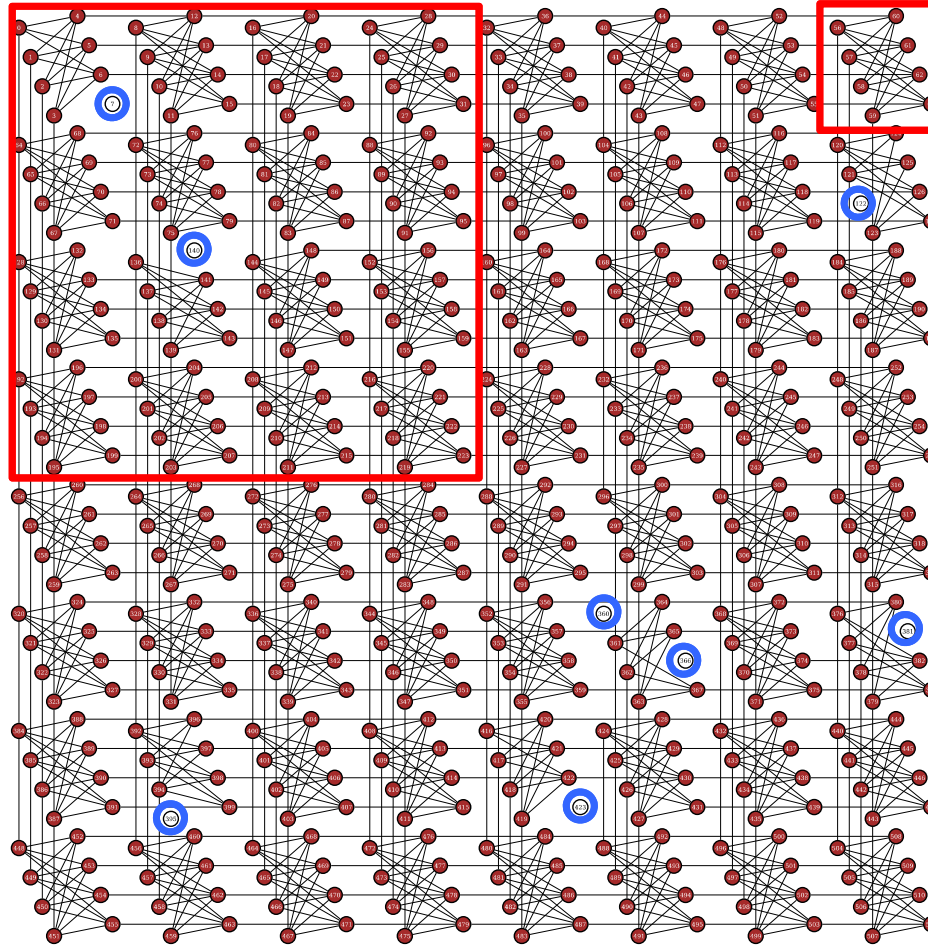


## Brief interlude on D-Wave processors

They are a type of quantum (& classical thermodynamics) simulator

# D-Wave 2 Connectivity Graph

512 qubits with "Chimera graph" couplings



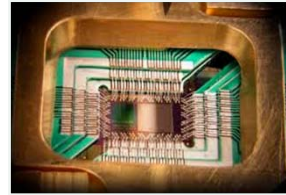
$K_{4,4}$  unit cell

Most "natural" problems defined over complete graphs; must be (minor-)embedded

Largest complete graph embeddable in  $L \times L$  Chimera graph of  $L$  unit cells, each  $K_{c,c}$  is  $K_{cL+1}$

All D-Wave chips to date:  $c = 4$   
Graph degree =  $c + 2 = 6$

$8 \times 8$  unit cells



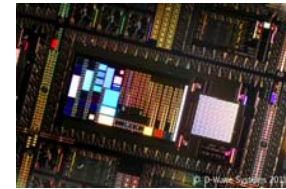
Oct 2011

**D-Wave 1:**

$L = 4, N=128$  (USC yield: 108)

$K_{17}$  for ideal

$K_{14}$  for actual



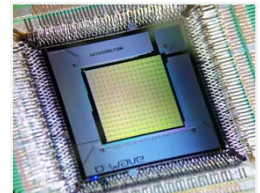
March 2013

**D-Wave 2:**

$L = 8, N=512$  (USC yield: 504)

$K_{33}$  for ideal

$K_{32}$  for actual



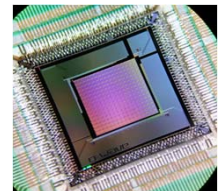
March 2016

**D-Wave 2X:**

$L = 12, N=1152$  (USC yield: 1098)

$K_{49}$  for ideal

$K_{44}$  for actual



Sep 2017

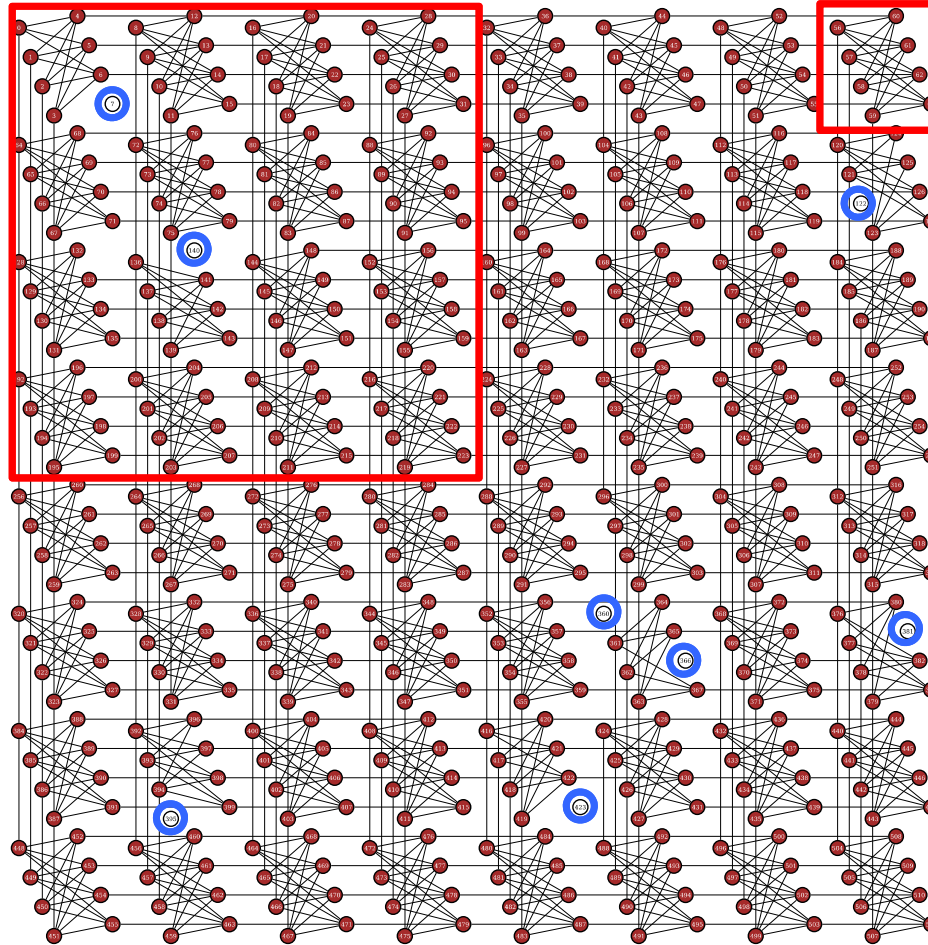
**D-Wave 2000Q**

$L = 16, N=2048$  (NASA yield: 2031)

$K_{65}$  for ideal

# D-Wave 2 Connectivity Graph

512 qubits with "Chimera graph" couplings



8 × 8 unit cells

←  $K_{4,4}$  unit cell

## D-Wave 1:

$L = 4, N = 128$  (USC yield: 108)

$K_{17}$  for ideal

$K_{14}$  for actual

## D-Wave 2:

$L = 8, N = 512$  (USC yield: 504)

$K_{33}$  for ideal

$K_{32}$  for actual

## D-Wave 2X:

$L = 12, N = 1152$  (USC yield: 1098)

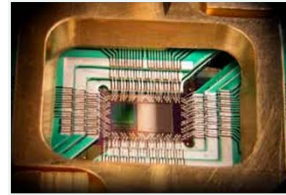
$K_{49}$  for ideal

$K_{44}$  for actual

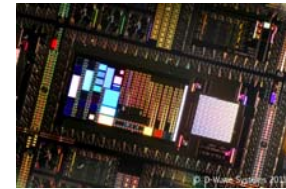
## D-Wave 2000Q

$L = 16, N = 2048$  (NASA yield: 2031)

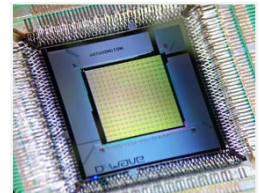
$K_{65}$  for ideal



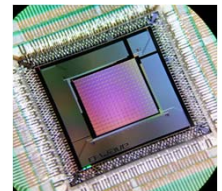
Oct 2011



March 2013



March 2016



Sep 2017

# Designed to solve/sample Ising model problems

Ideally, evolve adiabatically according to  $H(t) = A(t)H_X + B(t)H_P$

$$\begin{aligned}
 \downarrow H_X &= H_{\text{transverse}} = \sum_{j \in V} \sigma_j^x \\
 \uparrow H_P &= H_{\text{Ising}} = \sum_{j \in V} h_j \sigma_j^z + \sum_{(i,j) \in E} J_{ij} \sigma_i^z \sigma_j^z
 \end{aligned}$$

programmable

finds the ground state energy

$$\min_{\{\sigma_i^z\}} H_{\text{Ising}}$$

NP hard (Barahona 1982)

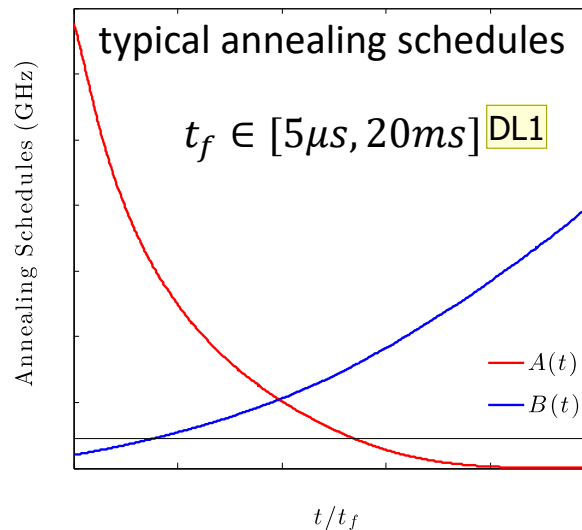
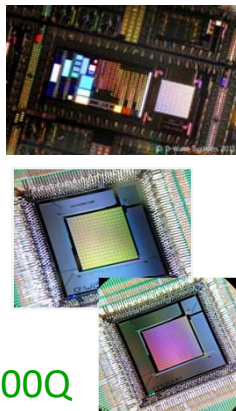
ideal, adiabatic quantum annealer

In reality: "control error"  
 $H$  implemented  $\neq H$  intended

D-Wave 2 precision:  
 $\delta h \sim N(0, 0.05 h_{\max})$ ,  
 $\delta J \sim N(0, 0.04 J_{\max})$   
 $\Rightarrow$  3 bits

... 4 bits on D-Wave 2X

... 5 bits on D-Wave 2000Q



Samples from Gibbs dist.

$$\frac{1}{Z} e^{-\beta H_{\text{Ising}}}$$

#P-hard (=counting)

$\sim 10\text{mK}$

## Diapositive 20

---

**DL1** Note colors are flipped!  
Daniel Lidar; 06/09/2017

## Beyond control errors: How quantum?

Facts:

- DW Ni flux qubits have  $T_2 \sim 100ns$ , annealing time  $t_f \geq 5\mu s$
- $\text{gap}(H)$  can be  $\ll T \sim 10mK$

Properly described as an open quantum system

**Governed by Markovian adiabatic master equation<sup>(1)</sup>**

- Dynamics (probably) not efficiently classically simulatable
- As an optimizer: so far no evidence of a q. speedup

A playground for testing algorithmic scaling with noisy qubits and error correction

<sup>(1)</sup> T. Albash, S. Boixo, DL, P. Zanardi, New J. of Phys. **14**, 123016 (2012); T. Albash, DL, PRA **91**, 062320 (2015)

# Error Correction for Quantum Annealing



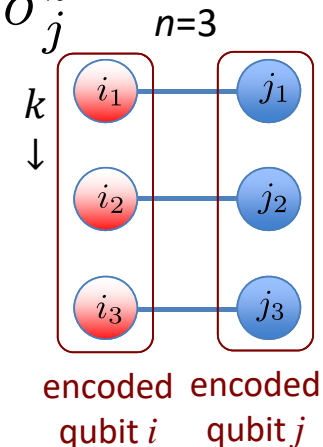
K. Pudenz, T. Albash & DL, Nature Comm. 5, 3243 (2014)

# Error Correction for Quantum Annealing

1. Encode into  
 $[n, 1, n]$   
 repetition code:

$$\overline{H}_{\text{Ising}} = \sum_{i=1}^N h_i \overline{\sigma}_i^z + \sum_{i < j}^N J_{ij} \overline{\sigma}_i^z \overline{\sigma}_j^z$$

$$\overline{\sigma}_i^z = \sum_{k=1}^n \sigma_{i_k}^z \quad \overline{\sigma}_i^z \overline{\sigma}_j^z = \sum_{k=1}^n \sigma_{i_k}^z \sigma_{j_k}^z$$



Nice features of this code:

**Implementable:** Logical Z and ZZ operators are 1 and 2-local

**Energy Boost:** Logical operators stronger than physical by factor of  $n$





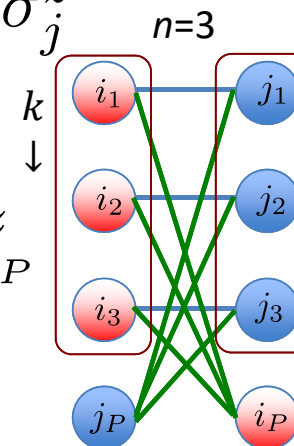
# Error Correction for Quantum Annealing

1. Encode into  
[n, 1, n]  
repetition code:

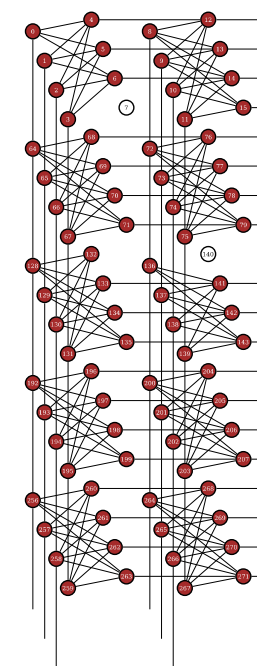
$$\overline{H}_{\text{Ising}} = \sum_{i=1}^N h_i \overline{\sigma}_i^z + \sum_{i < j}^N J_{ij} \overline{\sigma}_i^z \overline{\sigma}_j^z$$

2. Add FM  
energy penalty:

$$H_P = - \sum_{i=1}^N (\sigma_{i_1}^z + \dots + \sigma_{i_n}^z) \sigma_{i_P}^z$$



enforced by DW  
"Chimera"  
graph unit cell  
structure: tiling  
of  $K_{4,4}$ 's



**Also implementable:** ZZ operators are 2-local (stabilizers of the bit-flip repetition code)



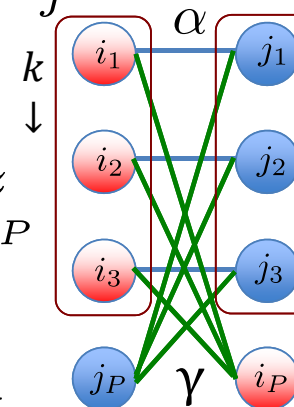
# Error Correction for Quantum Annealing

1. Encode into  
[n, 1, n]  
repetition code:

$$\overline{H}_{\text{Ising}} = \sum_{i=1}^N h_i \overline{\sigma}_i^z + \sum_{i < j}^N J_{ij} \overline{\sigma}_i^z \overline{\sigma}_j^z$$

2. Add FM  
energy penalty:

$$H_P = - \sum_{i=1}^N (\sigma_{i_1}^z + \dots + \sigma_{i_n}^z) \sigma_{i_P}^z$$



3. Combine:

$$\overline{H}_{\text{Ising},P}(\alpha, \gamma) := \underbrace{\alpha \overline{H}_{\text{Ising}}}_{\text{"problem scale" (controllable)}} + \underbrace{\gamma H_P}_{\text{"penalty scale" (optimized)}}$$



# Error Correction for Quantum Annealing

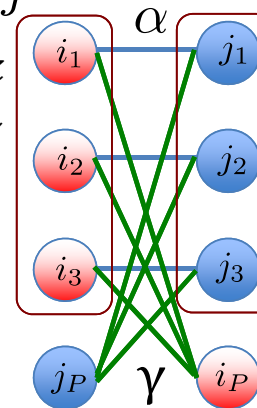
Quantum Annealing Correction (QAC)

1. Encode into  $[n, 1, n]$  repetition code:

$$\overline{H}_{\text{Ising}} = \sum_{i=1}^N h_i \overline{\sigma}_i^z + \sum_{i < j}^N J_{ij} \overline{\sigma}_i^z \overline{\sigma}_j^z$$

2. Add FM energy penalty:

$$H_P = - \sum_{i=1}^N (\sigma_{i_1}^z + \dots + \sigma_{i_n}^z) \sigma_{i_P}^z$$



3. Combine:

$$\overline{H}_{\text{Ising},P}(\alpha, \gamma) := \alpha \overline{H}_{\text{Ising}} + \gamma H_P$$

“problem scale”
“penalty scale”  
(controllable)
(optimized)

4. Run QA:

$$\overline{H}(t) = A(t) H_X + B(t) \overline{H}_{\text{Ising},P}(\alpha, \gamma)$$

5. Decode at end by majority vote

# Error Correction for Quantum Annealing

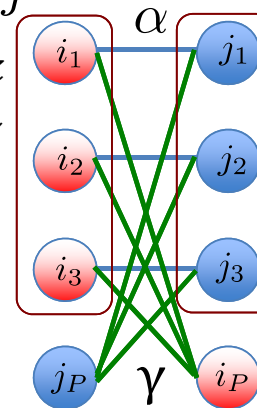
Quantum Annealing Correction (QAC)

1. Encode into  $[n, 1, n]$  repetition code:

$$\overline{H}_{\text{Ising}} = \sum_{i=1}^N h_i \overline{\sigma}_i^z + \sum_{i < j}^N J_{ij} \overline{\sigma}_i^z \overline{\sigma}_j^z$$

2. Add FM energy penalty:

$$H_P = - \sum_{i=1}^N (\sigma_{i_1}^z + \dots + \sigma_{i_n}^z) \sigma_{i_P}^z$$



3. Combine:

$$\overline{H}_{\text{Ising},P}(\alpha, \gamma) := \alpha \overline{H}_{\text{Ising}} + \gamma H_P$$

“problem scale”
“penalty scale”  
(controllable)
(optimized)

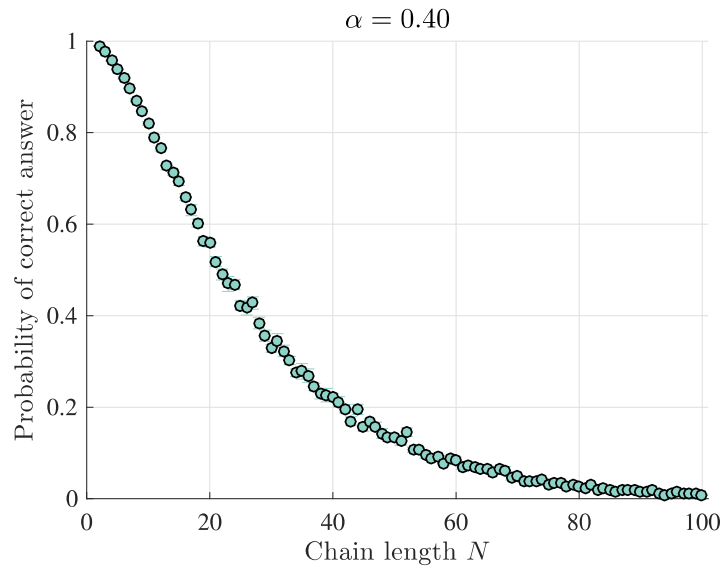
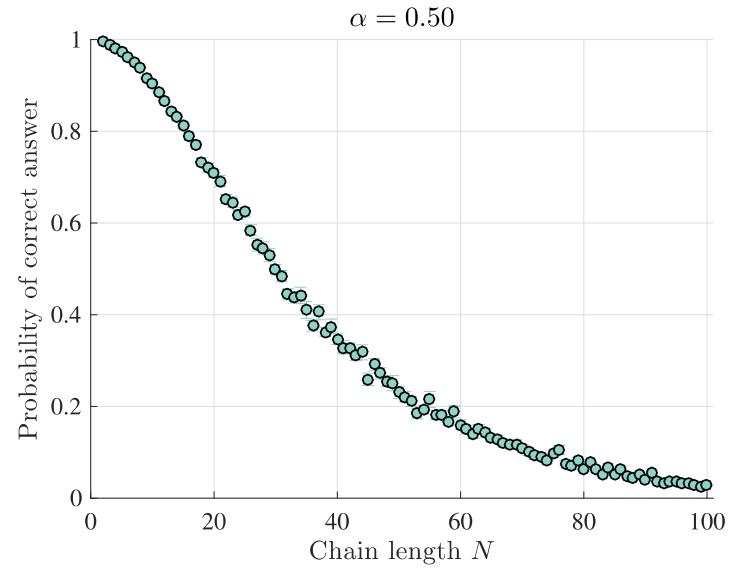
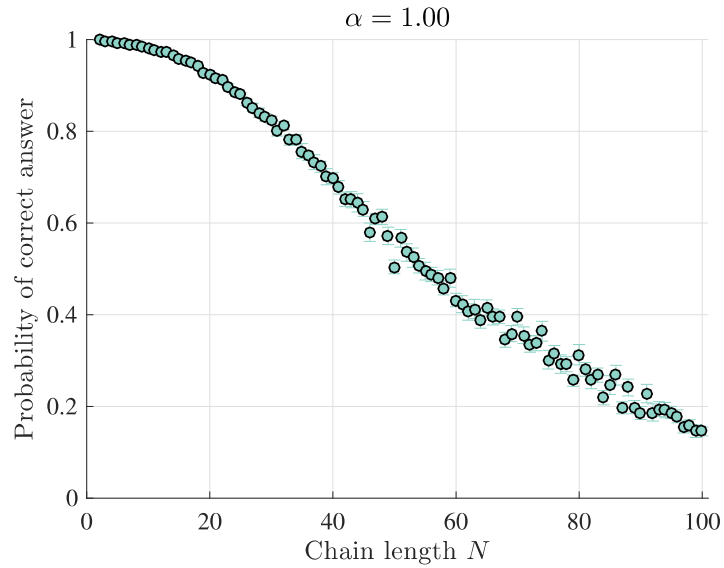
4. Run QA:

$$\overline{H}(t) = A(t) H_X + B(t) \overline{H}_{\text{Ising},P}(\alpha, \gamma)$$

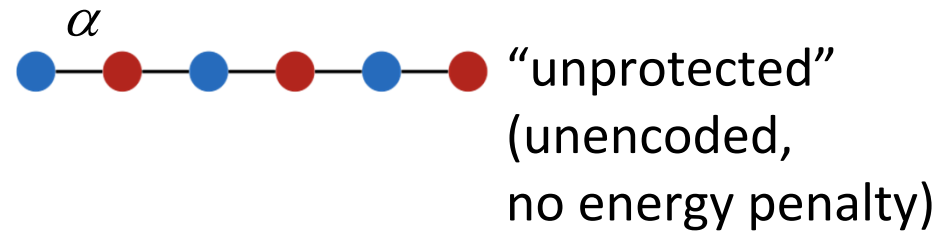
unencoded

5. Decode at end by majority vote

$$[\overline{\sigma}^x = (\sigma^x)^{\otimes n}]$$

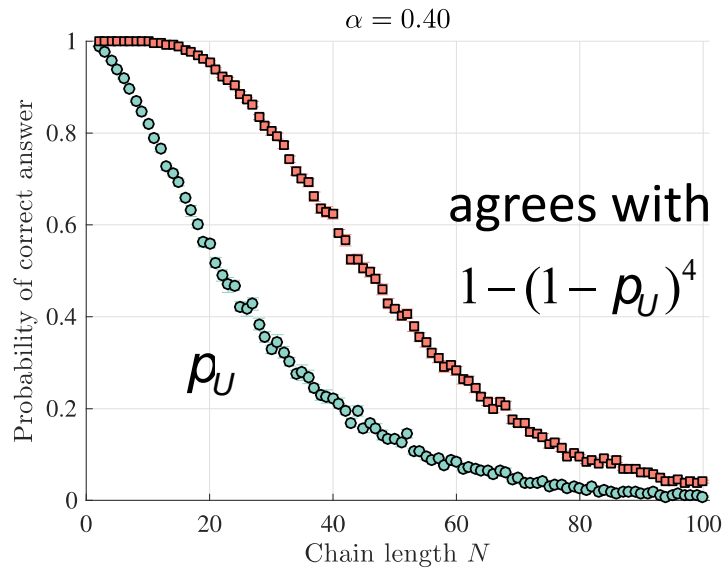
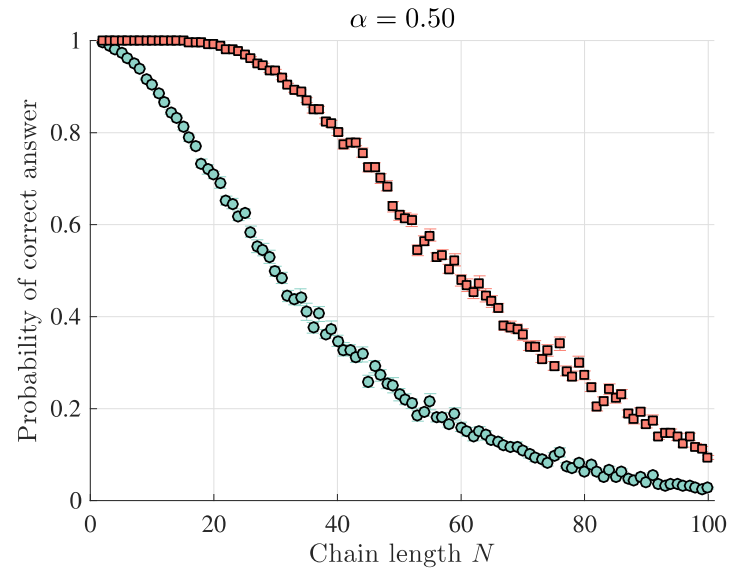
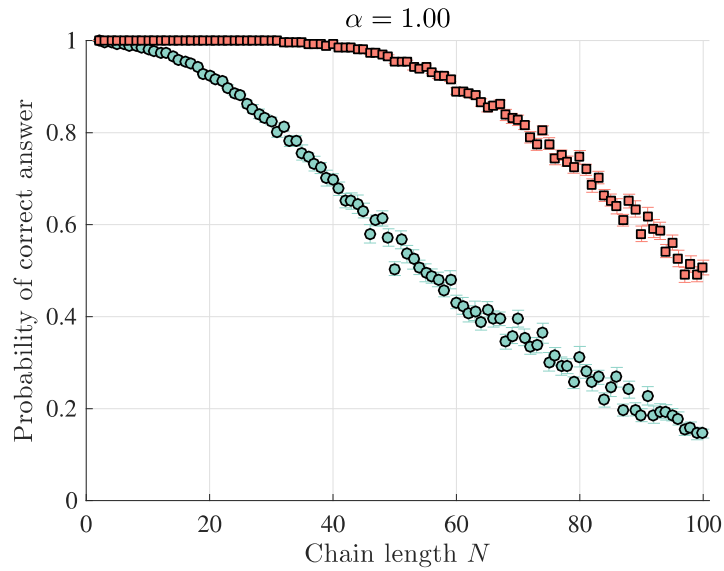


## test on AFM chains



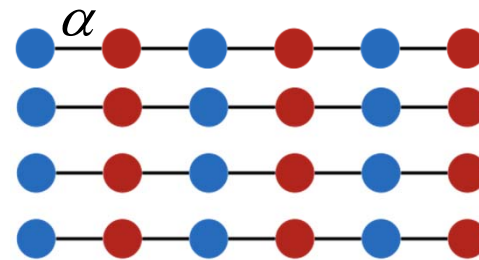
A. Mishra, T. Albash & DL, Q. Info. Proc. (2016)  
K. Pudenz, T. Albash & DL, Nat. Comm. (2014)

○ Unprotected(U)

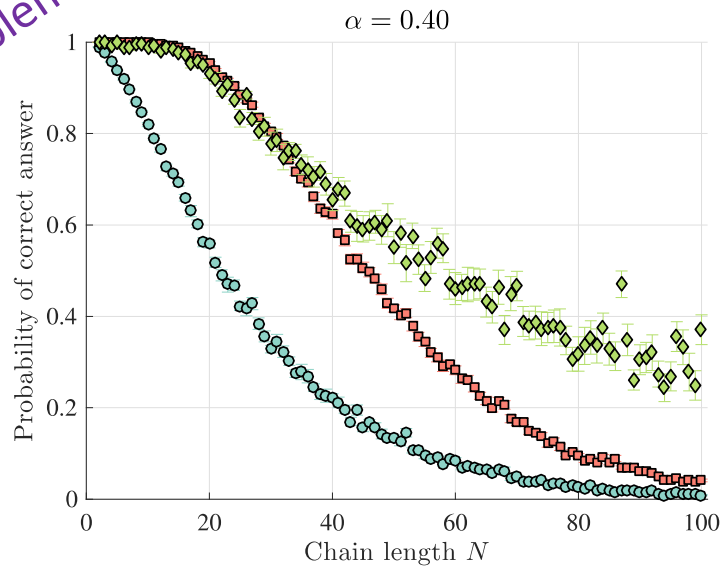
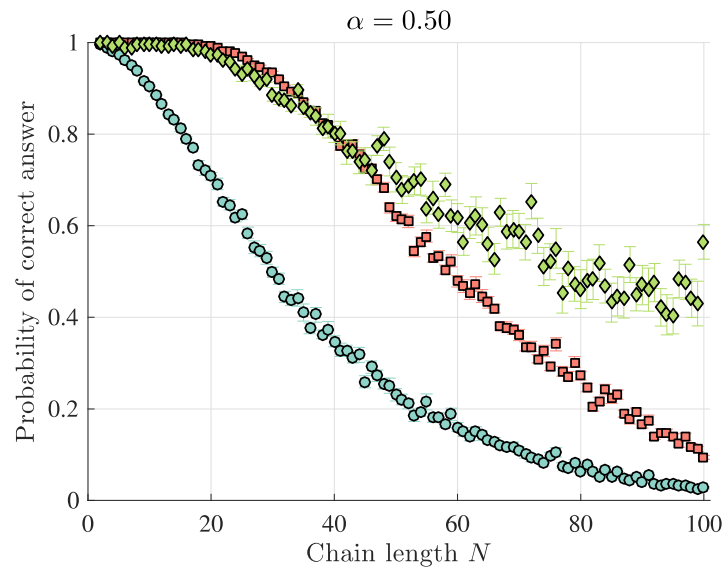
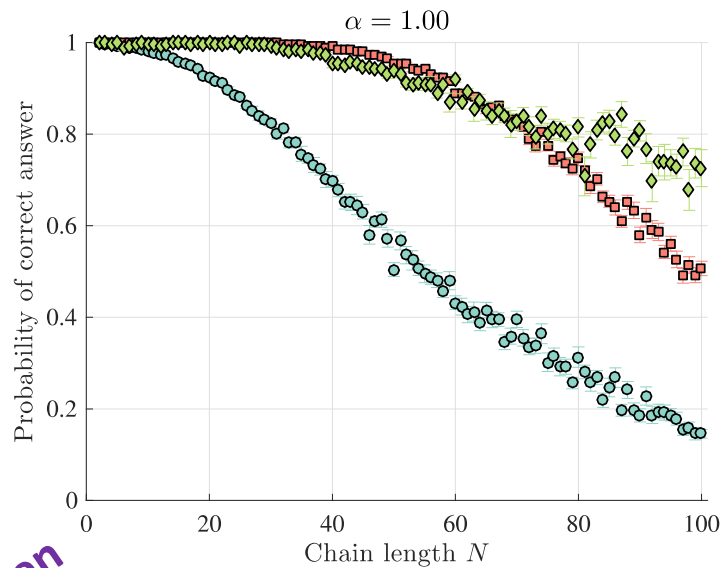


● Unprotected(U)    ■ Classical(C)

Fair comparison for QAC:  
run 4 parallel chains, take the best  
("classical" error correction)



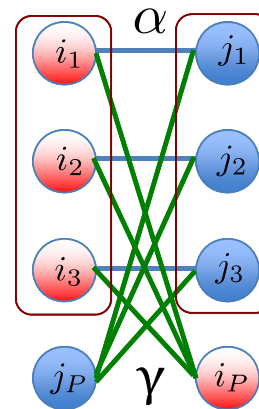
A. Mishra, T. Albash & DL, Q. Info. Proc. (2016)  
K. Pudenz, T. Albash & DL, Nat. Comm. (2014)



algorithmic breakeven  
(for a trivial problem)

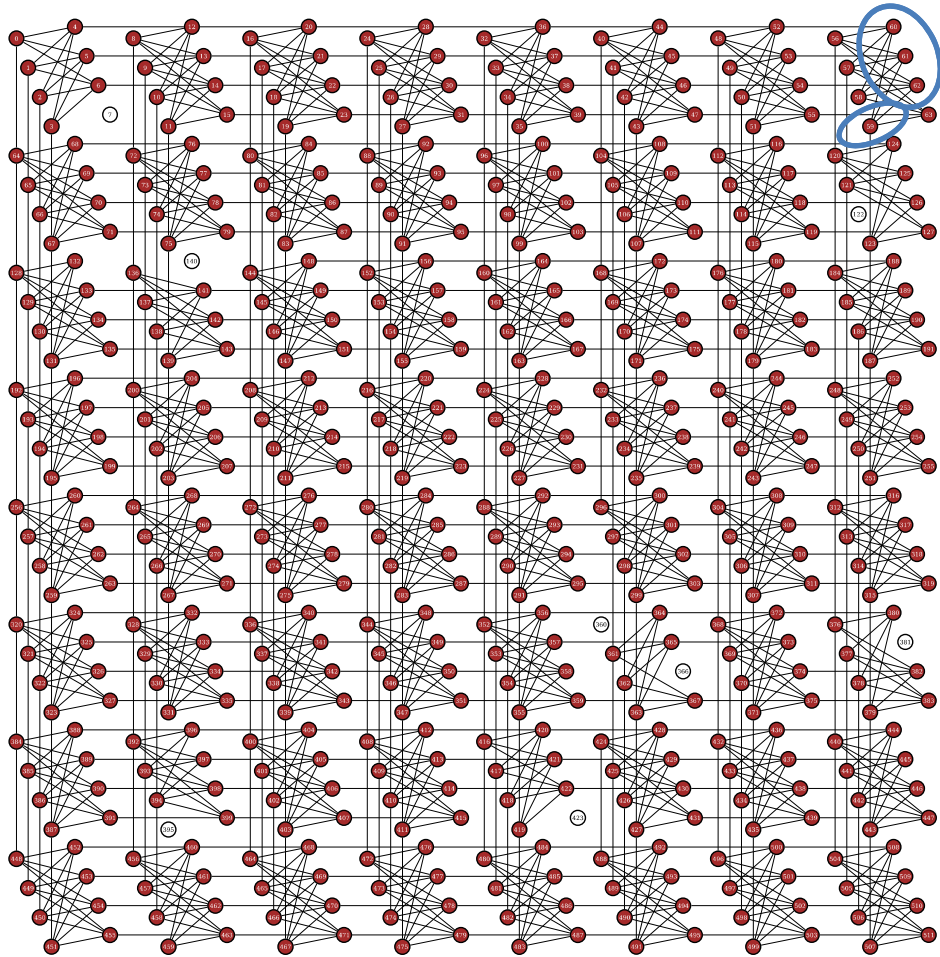
○ Unprotected(U)    □ Classical(C)    ◆ QAC

QAC

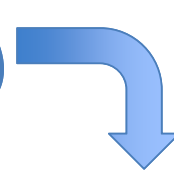


A. Mishra, T. Albash & DL, Q. Info. Proc. (2016)  
K. Pudenz, T. Albash & DL, Nat. Comm. (2014)

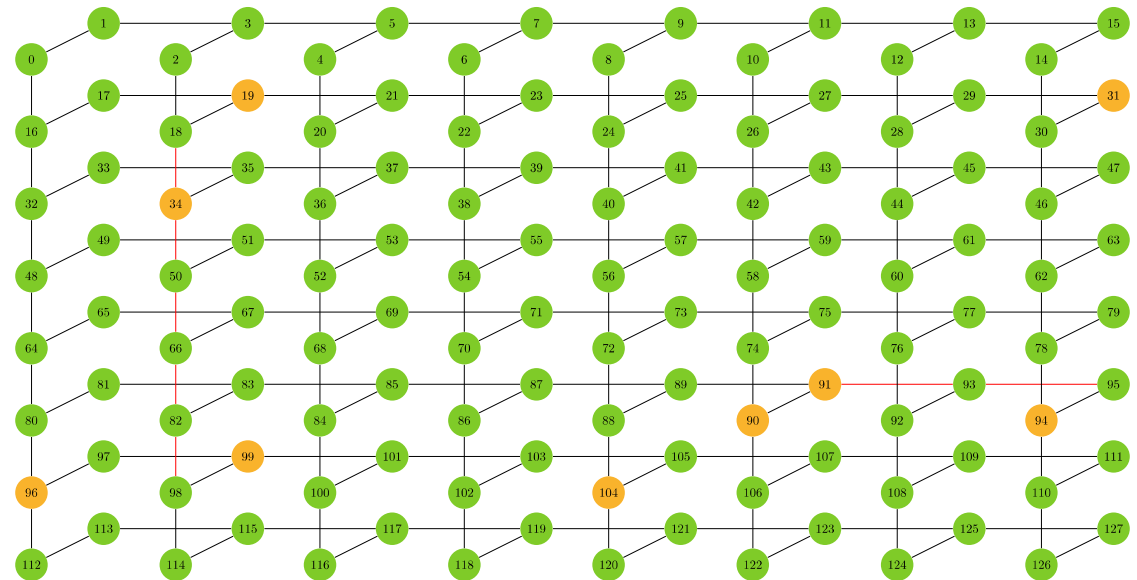
# From chains to random non-planar Ising ...



Chimera(DW2); degree 6; 503 functional qubits



QAC encoding



degree 3; still non-planar  
119 functional logical qubits

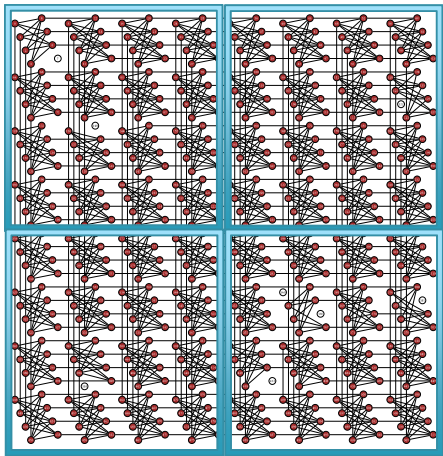
$$H_{\text{Ising}} = \sum_{(i,j) \in E(G)} J_{ij} \overline{\sigma_i^z} \overline{\sigma_j^z}$$

$$\text{random } J_{ij} \in \left\{ \pm \frac{1}{6}, \dots, \pm \frac{5}{6}, \pm 1 \right\}$$



# Algorithmic breakeven with quantum annealing correction

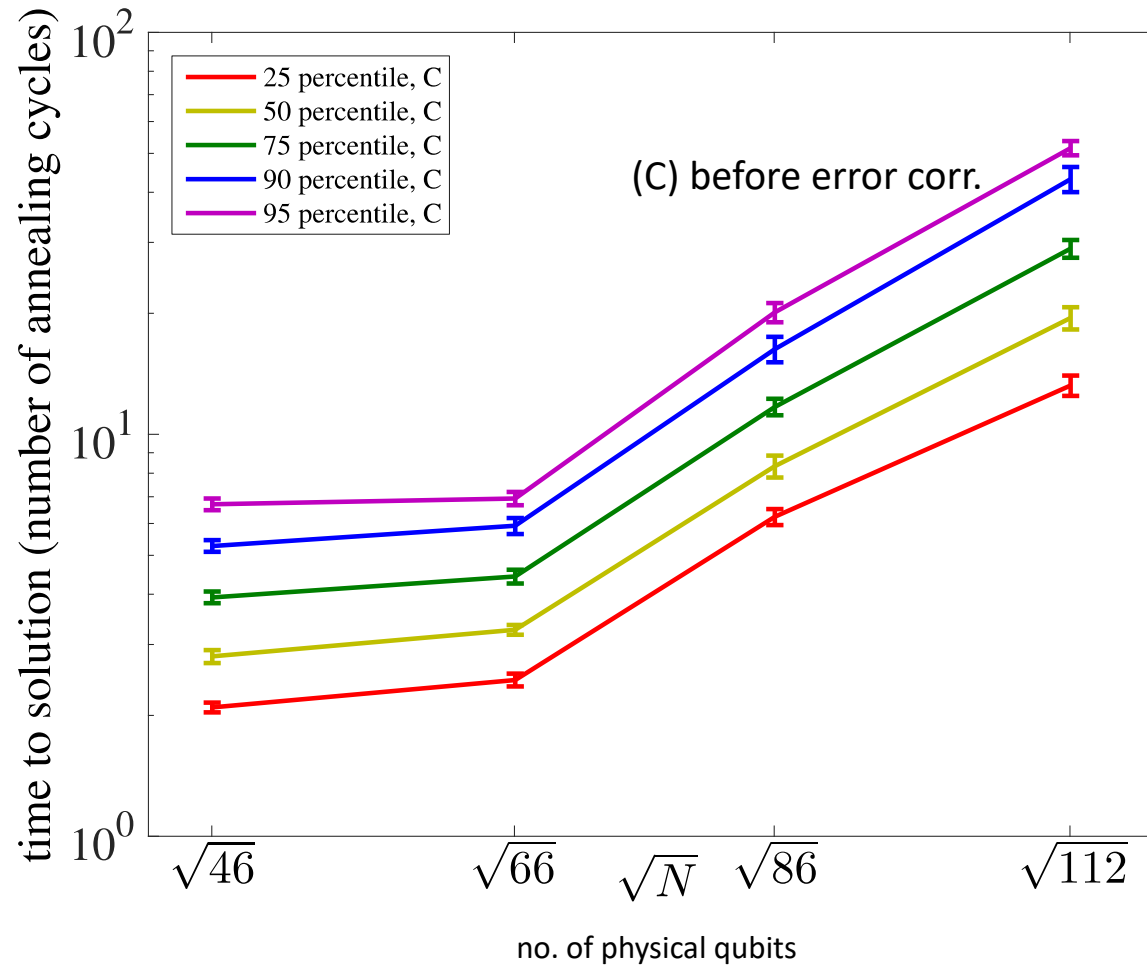
“classical” (C)



best of 4 copies

$$H_{\text{Ising}} = \sum_{(i,j) \in E(G)} J_{ij} \sigma_i^z \sigma_j^z$$

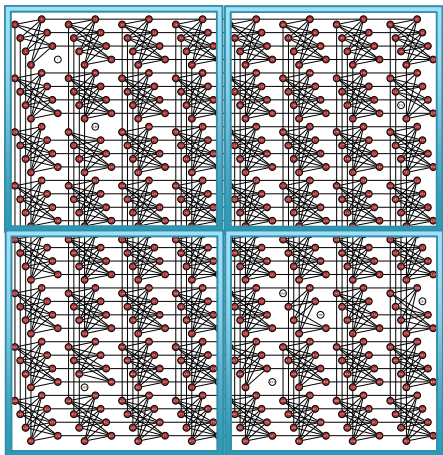
$$J_{ij} \in \left\{ \pm \frac{1}{6}, \dots, \pm \frac{5}{6}, \pm 1 \right\}$$



(C) before error corr.

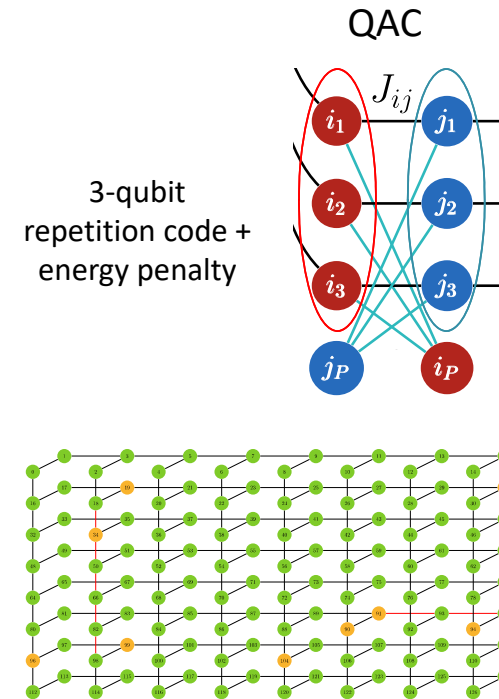
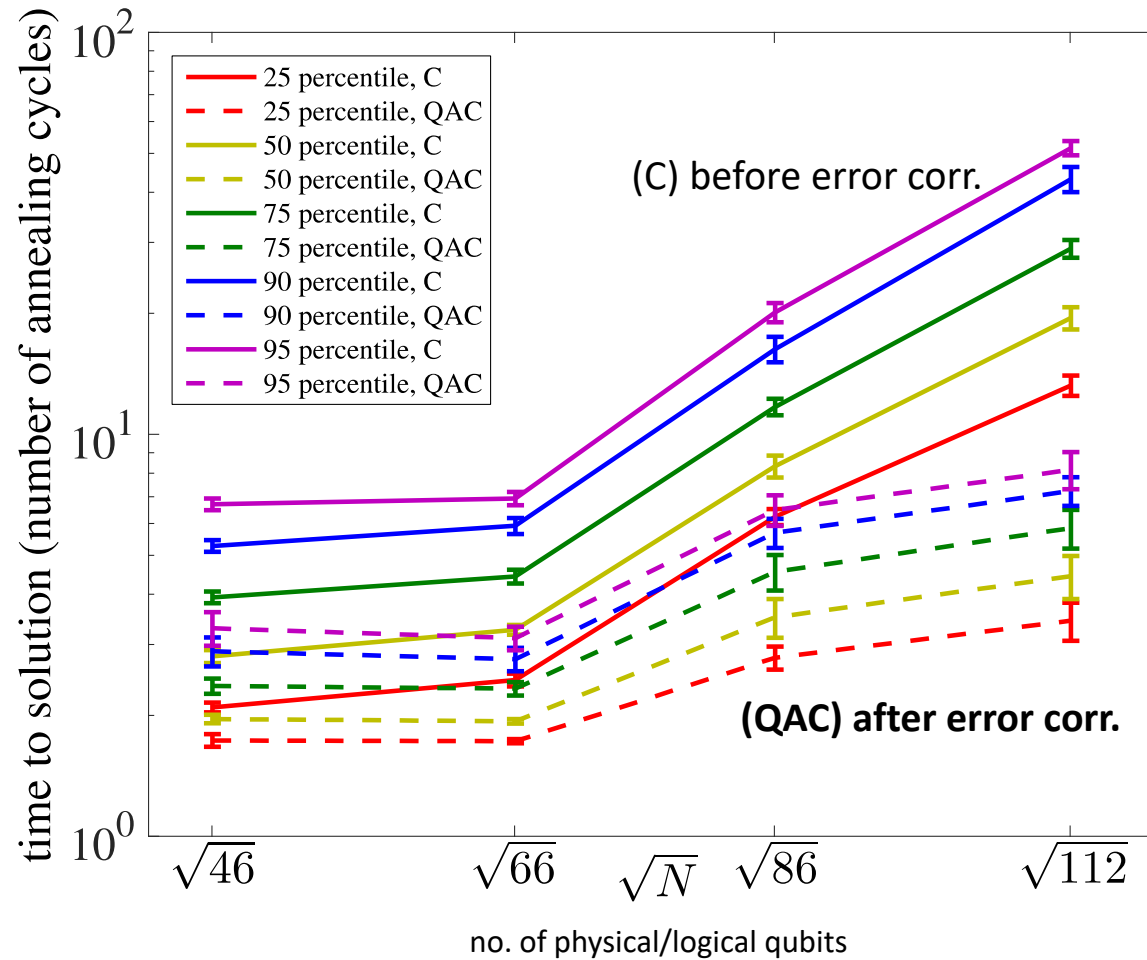
# Algorithmic breakeven with quantum annealing correction

“classical” (C)



best of 4 copies

$$H_{\text{Ising}} = \sum_{(i,j) \in E(G)} J_{ij} \sigma_i^z \sigma_j^z$$



$$H_{\text{Ising}} = \sum_{(i,j) \in E(G)} J_{ij} \sigma_i^z \sigma_j^z$$

$$J_{ij} \in \left\{ \pm \frac{1}{6}, \dots, \pm \frac{5}{6}, \pm 1 \right\}$$

# Why does QAC work?

Main mechanism:

avoidance or modification of a quantum phase transition  
due to penalty term



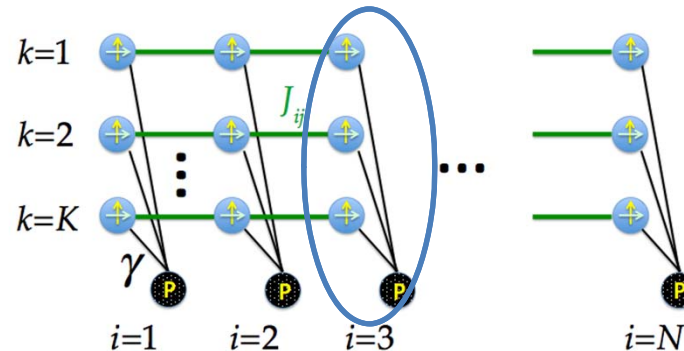
S. Matsuura, H. Nishimori, W. Vinci, T. Albash, DL. Phys. Rev. A 95, 022308 (2017)

S. Matsuura, H. Nishimori, T. Albash. DL., Phys. Rev. Lett. 116, 220501 (2016)

# Why does QAC work?

## Mean-field analysis of $p$ -body ferromagnet

$N$  logical qubits  
each in  $[K, 1, K]$   
repetition code  
( $K = n$  in earlier notation)



Model Hamiltonian  $H = H_x + H_z$

$$H_z = \underbrace{-N \sum_{k=1}^K \left( \frac{1}{N} \sum_{i=1}^N \sigma_{iz}^k \right)^p}_{\text{problem Hamiltonian}} - \gamma \sum_{k=1}^K \sum_{i=1}^N \sigma_{iz}^k \sigma_{iz}^0$$

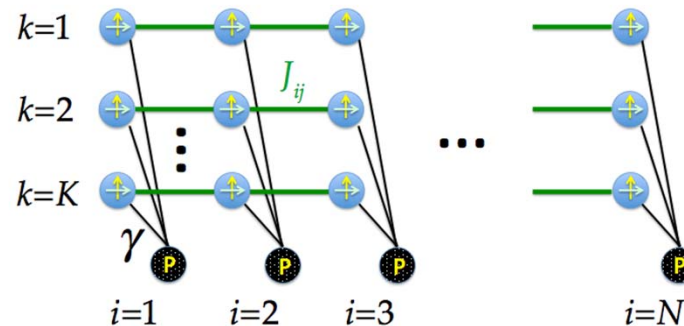
problem Hamiltonian  
fully-connected logical qubits

penalty

# Why does QAC work?

## Mean-field analysis of $p$ -body ferromagnet

$N$  logical qubits  
each in  $[K, 1, K]$   
repetition code  
( $K = n$  in earlier notation)



Model Hamiltonian  $H = H_x + H_z$

$$H_z = -N \sum_{k=1}^K \left( \frac{1}{N} \sum_{i=1}^N \sigma_{iz}^k \right)^p - \gamma \sum_{k=1}^K \sum_{i=1}^N \sigma_{iz}^k \sigma_{iz}^0$$

$$H_x = -\Gamma \sum_{k=1}^K \sum_{i=1}^N \sigma_{ix}^k$$

annealing schedule

transverse Hamiltonian

penalty

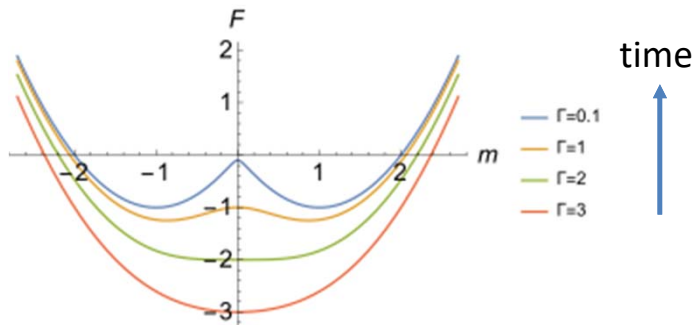
use stat. mech. tricks to  
compute partition function

$$Z = \text{Tr} e^{-\beta(H_z + H_x)}$$

then free energy etc.

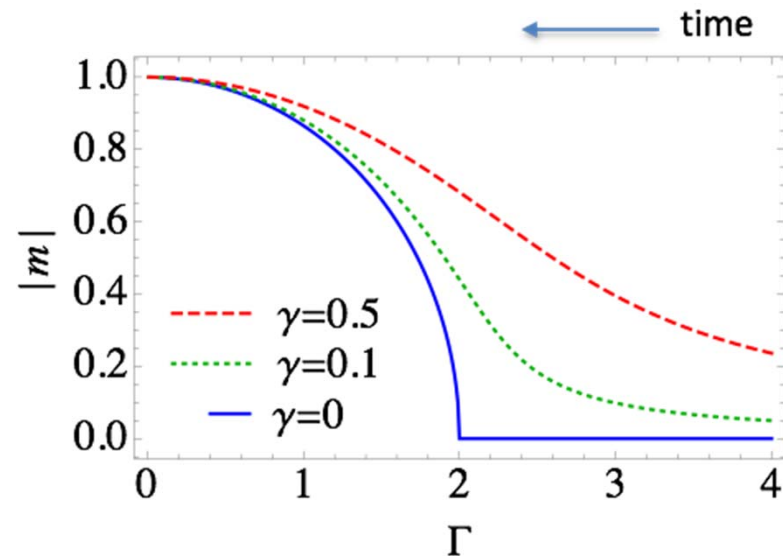
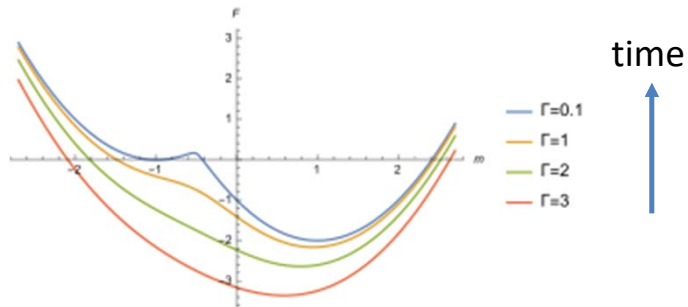
# Free energy and magnetization for $p=2$

$\gamma = 0$



no penalty:  
2nd order phase transition

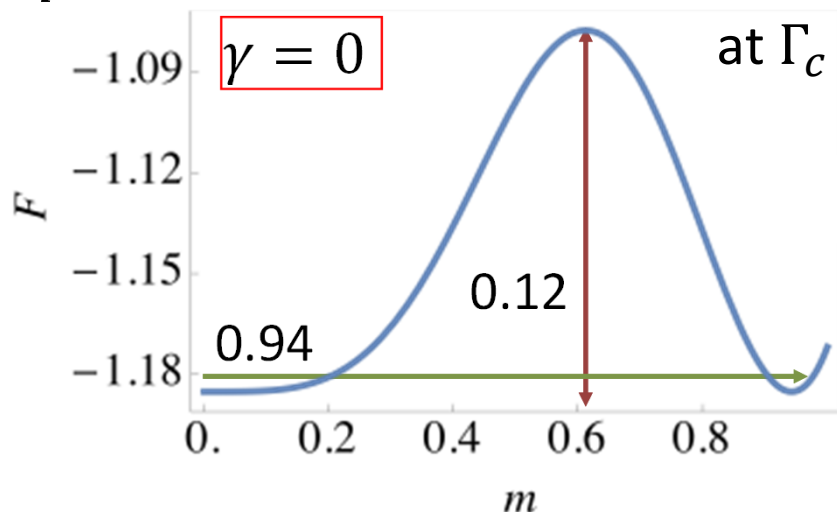
$\gamma = 1$



Turning the penalty on *avoids* the phase transition

# 1st order transition for $p \geq 3$

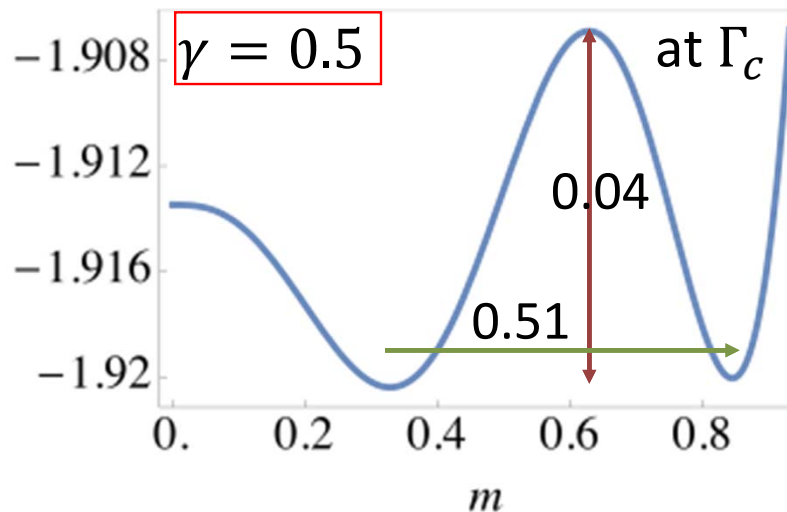
$p = 4$



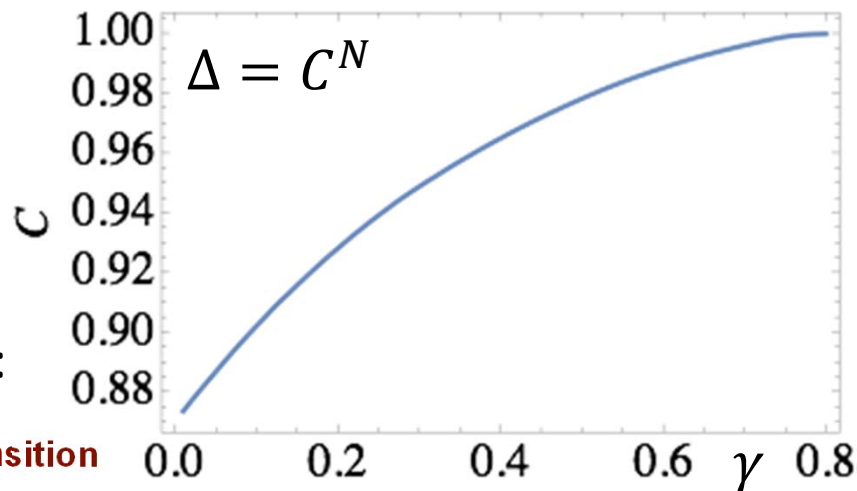
discontinuous transition from  $m=0$  to  $m=0.94$   
(wide and tall tunneling barrier)

Increasing penalty  $\gamma$   
lowers and narrows  
the tunneling barrier

also increases the minimum gap  $\Delta$ :



discontinuous transition from  $m=0.33$  to  $m=0.84$



Turning the penalty on softens the phase transition

## Why does QAC work?

- For models with a **second** order quantum phase transition:
  - QAC **avoids the phase transition**
- For models with a **first** order quantum phase transition:
  - QAC **softens the closing of the gap**

S. Matsuura, H. Nishimori, T. Albash. DL., Phys. Rev. Lett. 116, 220501 (2016)

S. Matsuura, H. Nishimori, W. Vinci, T. Albash, DL. Phys. Rev. A 95, 022308 (2017)

- QAC **rescales the temperature**; rescaling can be made as large as  $n^2$  for  $[n, 1, n]$  code (using all-to-all connectivity):
  - qubits can be traded for temperature reduction**

W. Vinci, T. Albash & DL, Nature Quant. Info. 2, 16017 (2016)

➔ Helps to **achieve algorithmic breakeven**

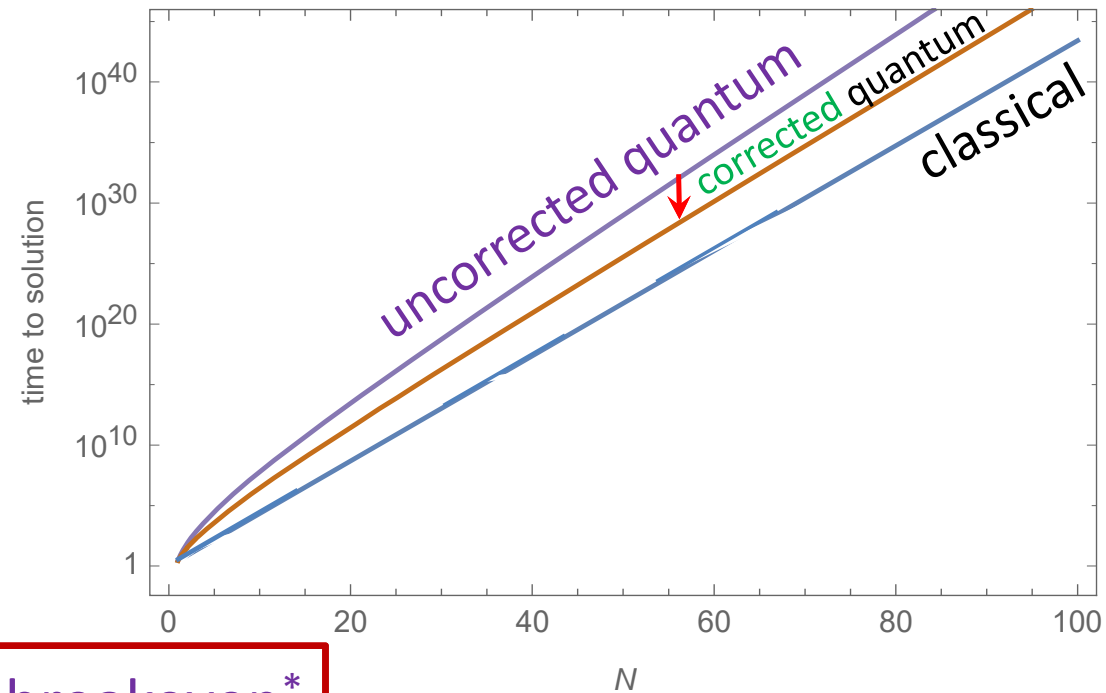


∴ Target for Error-Corrected Circuit-Model Quantum Computing

achieve algorithmic breakeven\*

\*Demonstrate error-corrected scaling that is no worse than uncorrected on a computational problem

# ∴ Target for Error-Corrected Circuit-Model Quantum Computing

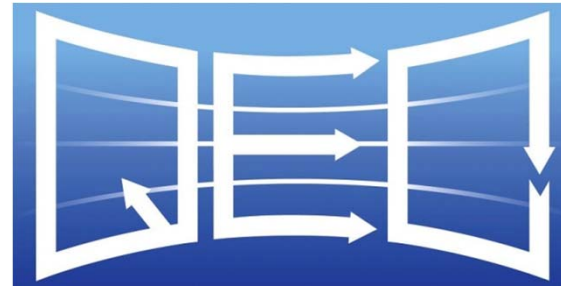


achieve algorithmic breakeven\*

# Thanks!

\* Demonstrate error-corrected scaling that is no worse than uncorrected on a computational problem

# The Future



As of June 2017, funded under IARPA's **"Quantum Enhanced Optimization"** (QEO) Program we will implement all the lessons we've learned studying the D-Wave machines, with a mission to build a 100-qubit quantum annealer in 5 years that is



Additional slides

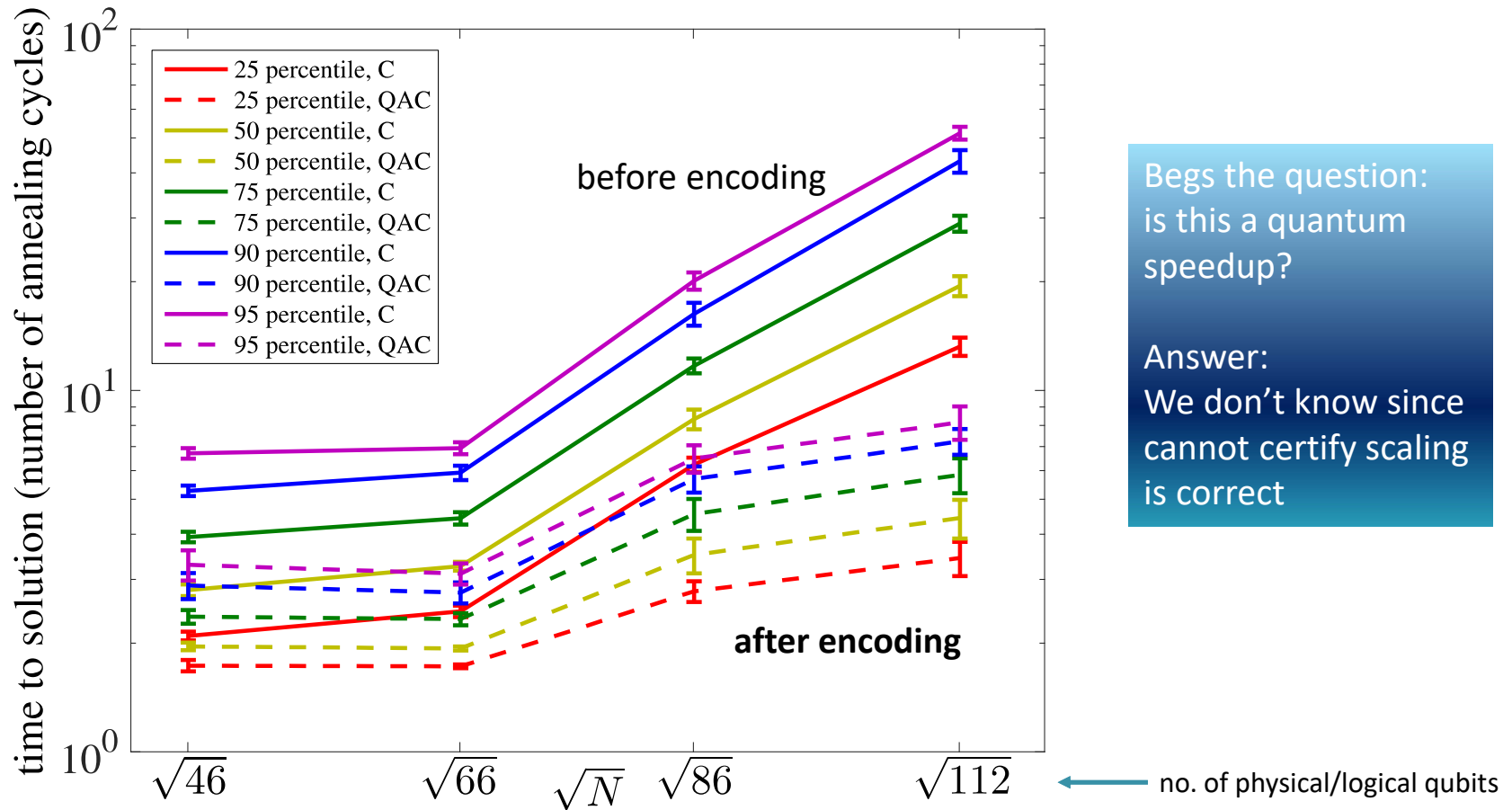
# Surpassing breakeven with quantum annealing correction

$$H_{\text{Ising}} = \sum_{j \in V} h_j \sigma_j^z + \sum_{(i,j) \in E} J_{ij} \sigma_i^z \sigma_j^z$$

random Ising

$$h_j = 0$$

$$J_{ij} \in \left\{ \pm \frac{1}{6}, \dots, \pm \frac{5}{6}, \pm 1 \right\}$$



Begs the question:  
is this a quantum  
speedup?

Answer:  
We don't know since  
cannot certify scaling  
is correct

## Certi fiable speedup requires a proof of optimality

**Time-to-solution (TTS):**

the time required to find the ground state (GS) at least once  
with probability 99%

Run the device with annealing time  $t_a$  and measure probability  $p_{\text{GS}}$  of finding the GS. Then:

$$\text{TTS} = t_a \times (\text{no. of repetitions to get to 99\%}) = t_a \frac{\log(1-0.99)}{\log(1-p_{\text{GS}}(t_a))}$$

The **optimal annealing time**  $t_a$  is that  
which **minimizes** the TTS for fixed problem size  $N$

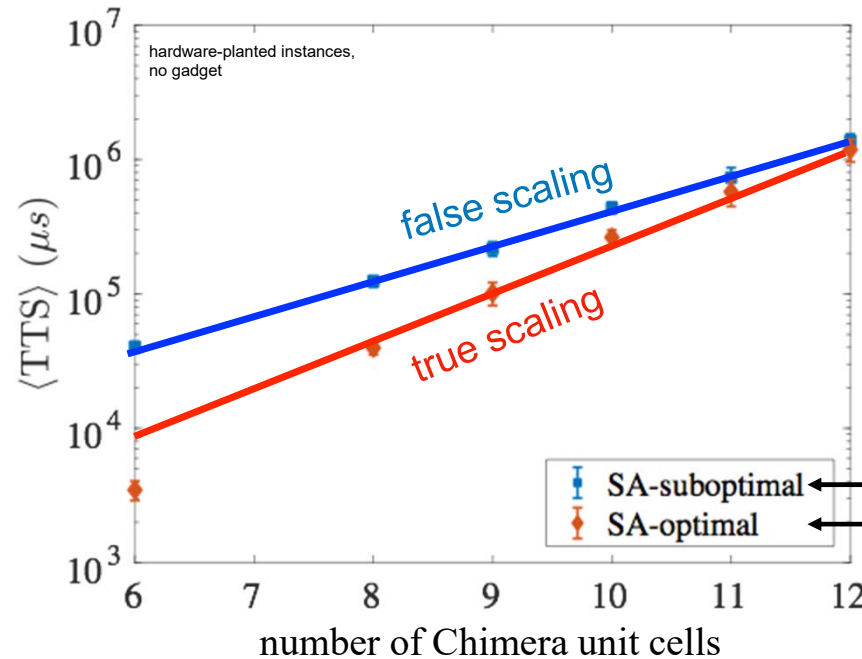
*Scaling cannot be trusted unless optimal  $t_a$  has been identified<sup>(1,2)</sup>*

(1) T.F. Rønnow, Z. Wang, J. Job, S. Boixo, S.V. Isakov, D. Wecker, J.M. Martinis, D.A. Lidar, and M. Troyer, Science 345, 420 (2014)

(2) I. Hen, J. Job, T. Albash, T.F. Rønnow, M. Troyer, D.A. Lidar, Phys. Rev. A 92, 042325 (2015)

# Certifiable speedup requires a proof of optimality

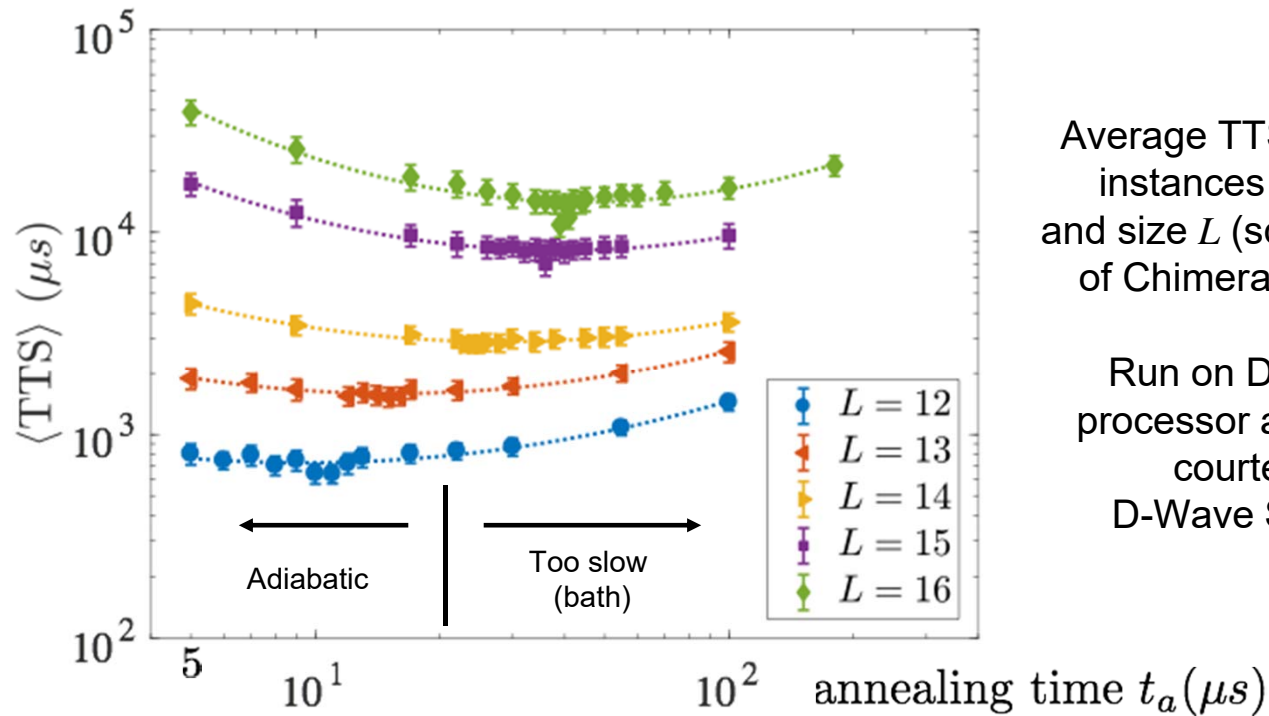
Excessively long annealing time causes **false flat scaling**



Scaling curves without an optimality certificate cannot be trusted

# An optimal annealing time certificate

Smoking gun is a **minimum** in TTS as a function of annealing time ...



Average TTS over 1000 instances at each  $t_a$  and size  $L$  (sqrt of number of Chimera unit cells).

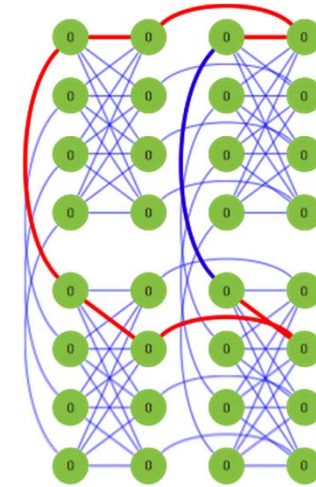
Run on DW2000Q processor at Burnaby, courtesy of D-Wave Systems.

... for a new problem class: 'logical-planted-solution instances'



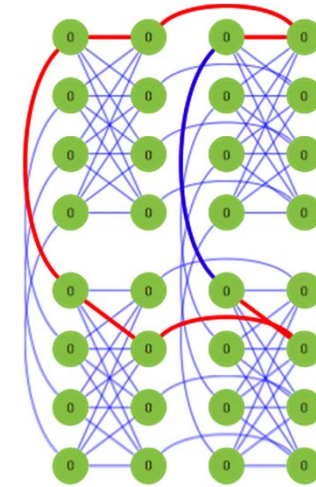
# `logical-planted-solution' instances

1. Create a spin-glass backbone using frustrated loops with planted solutions<sup>(1)</sup> on the **logical graph** of Chimera unit cells<sup>(2)</sup>

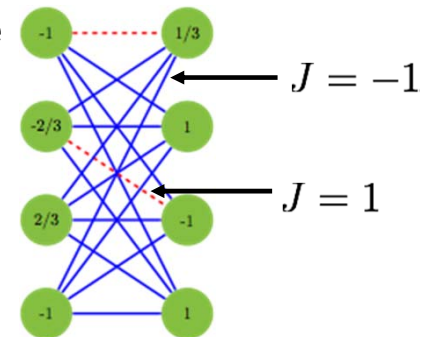


# 'logical-planted-solution' instances

1. Create a spin-glass backbone using frustrated loops with planted solutions on the **logical graph** of Chimera unit cells



2. Add 8-qubit 'gadgets' placed randomly in some fraction (10%) of all unit cells



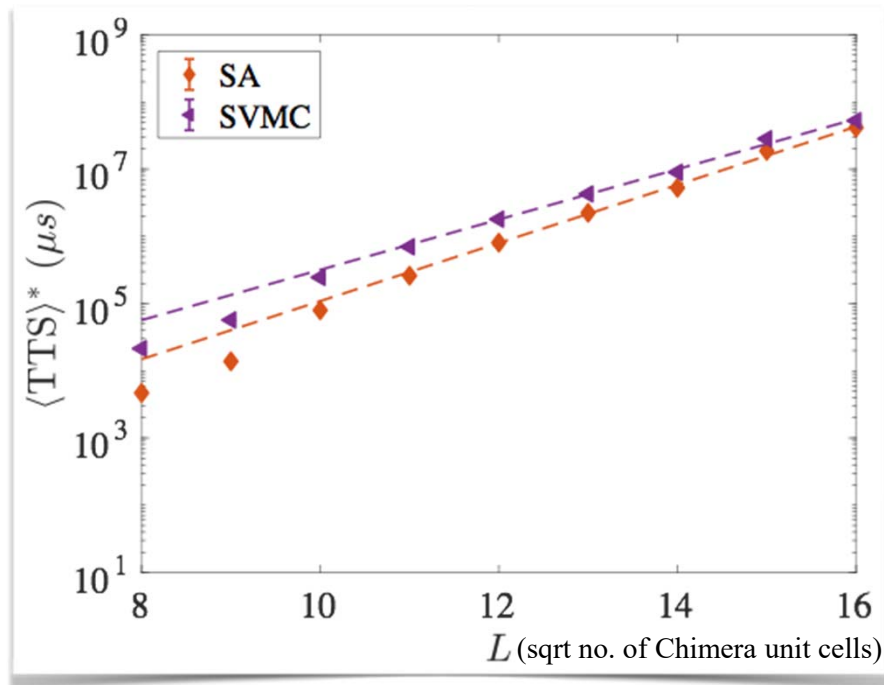
Can now use to demonstrate **limited** quantum speedup

# Limited quantum speedup: the evidence

Speedup relative to a 'relevant' class of classical algorithms

Start with two optimized 'DW-like' classical solvers:

- Simulated Annealing (SA) with single-spin updates
- Spin-vector Monte Carlo<sup>(1)</sup> (SVMC)



Fit curves to  
 $a \exp(bL)$

Solver	$b$ [95% CI]
SA	$1.002 \pm 0.066$
SVMC	$0.867 \pm 0.079$

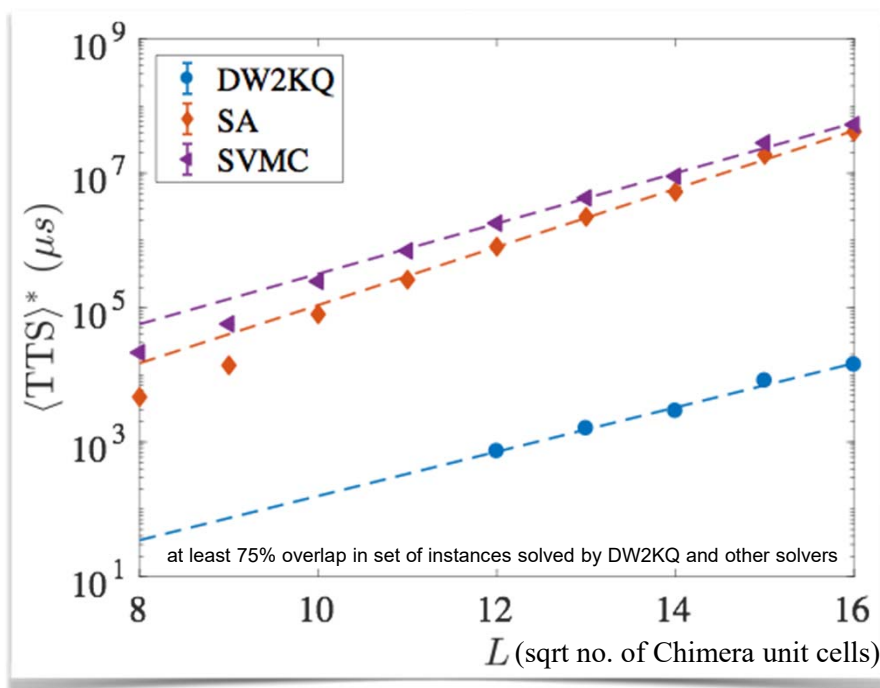
scaling of the median of the instance distribution

(1) S. Shin, G. Smith, J. Smolin, U. Vazirani, arXiv:1401.7087

# Limited quantum speedup: the evidence

Speedup relative to a 'relevant' class of classical algorithms

DW2000Q unequivocally beats the classical solvers



Fit curves to  
 $a \exp(bL)$

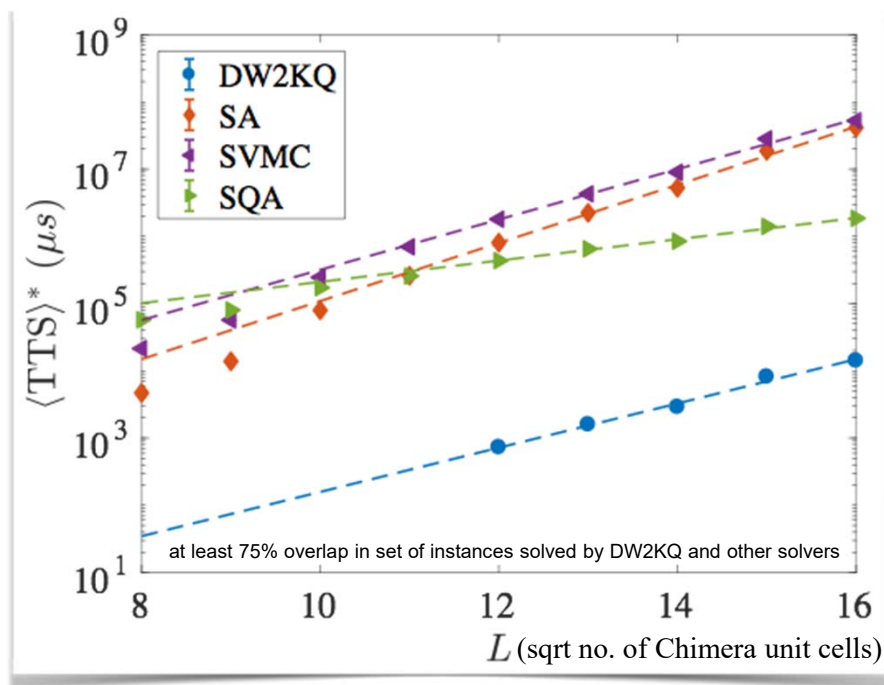
Solver	$b$ [95% CI]
DW2KQ	$0.760 \pm 0.017$
SA	$1.002 \pm 0.066$
SVMC	$0.867 \pm 0.079$

scaling of the median of the instance distribution

# Limited quantum speedup: the evidence

Speedup relative to a `relevant' class of classical algorithms

Simulated quantum annealing<sup>(1)</sup> dominates



Fit curves to  
 $a \exp(bL)$

Solver	$b$ [95% CI]
DW2KQ	$0.760 \pm 0.017$
SA	$1.002 \pm 0.066$
SVMC	$0.867 \pm 0.079$
SQA	$0.370 \pm 0.052$

scaling of the median of the instance distribution

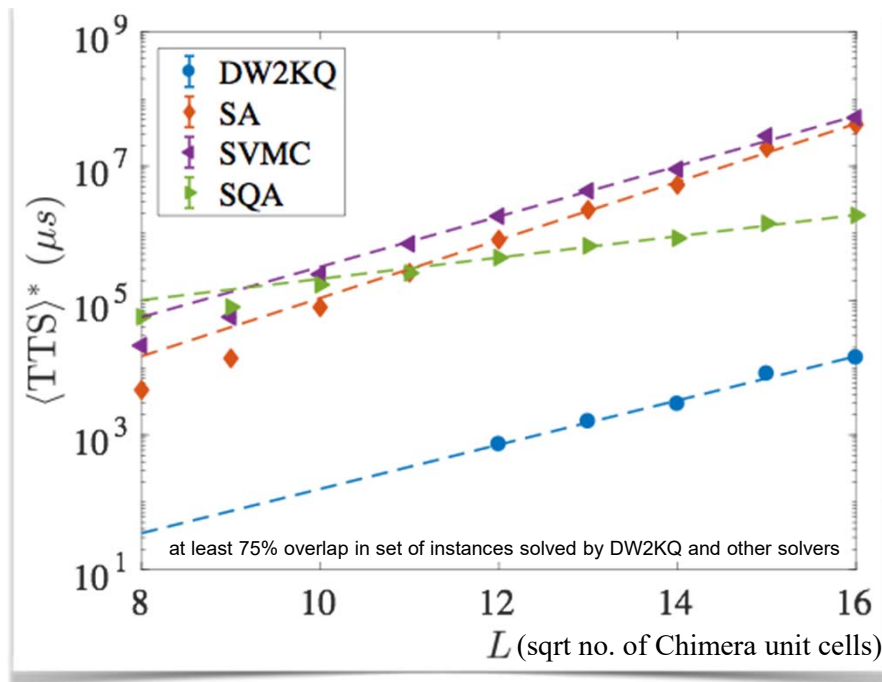
(1) G.E. Santoro, R. Martoňák, E. Tosatti and R. Carr, Science 295, 2427 (2002) [discrete-time path integral Monte Carlo]

# Limited quantum speedup: the evidence

Speedup relative to a `relevant' class of classical algorithms

Simulated quantum annealing<sup>(1)</sup> dominates

Can we reach breakeven (and beyond) through quantum annealing correction?



Fit curves to  
 $a \exp(bL)$

Solver	$b$ [95% CI]
DW2KQ	$0.760 \pm 0.017$
SA	$1.002 \pm 0.066$
SVMC	$0.867 \pm 0.079$
SQA	$0.370 \pm 0.052$

scaling of the median of the instance distribution

(1) G.E. Santoro, R. Martoňák, E. Tosatti and R. Carr, Science 295, 2427 (2002)

Backup slides

# Error/Noise Sources

## ❖ Technical/Engineering:

- Non-ideal spin implementation (not a true 2-level system)
- Finite digital-to-analog conversion (DAC) precision
- Imperfect device characterization & calibration
- Crosstalk
- Low-frequency flux noise

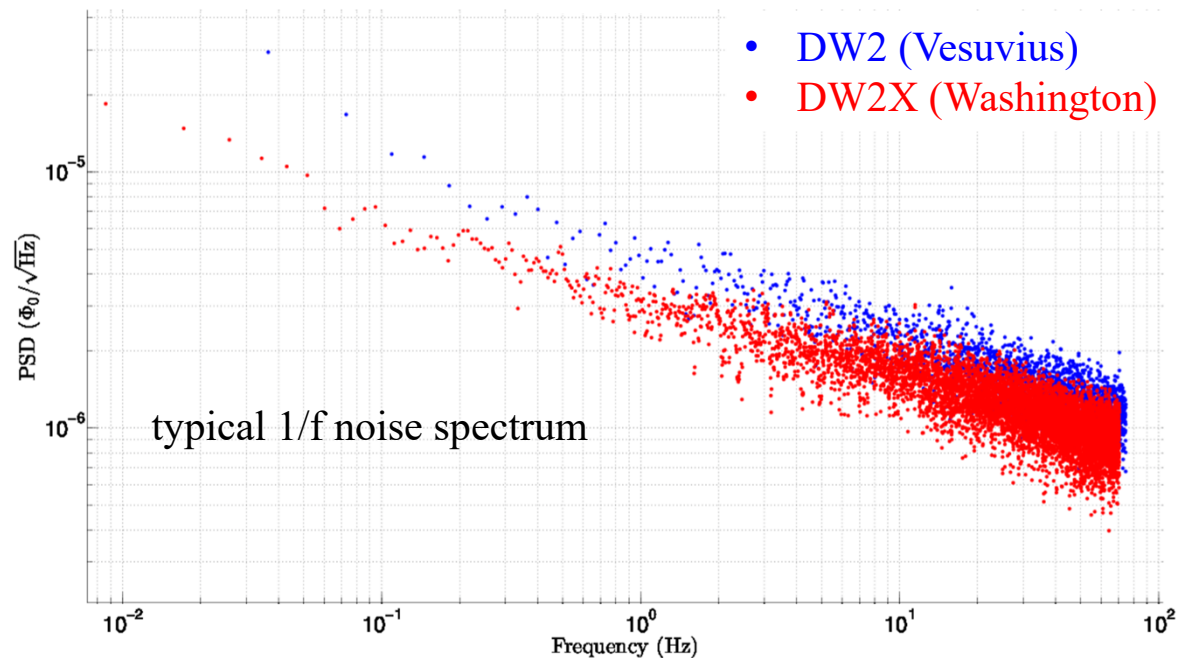
$h_i/J_{ij}$  control:

DW2: 3 bits precision

DW2X: 4 bits precision

## ❖ Decoherence:

- Thermal excitations ( $T_1$ )
- Dephasing in computational basis ( $T_2$ ; problematic for instances with small gap)
- Dephasing between energy eigenstates





## Adiabatic Markovian Master Equation

T. Albash, S. Boixo, DL, P. Zanardi, New J. of Phys. **14**, 123016 (2012); T. Albash, DL, PRA **91**, 062320 (2015)

Weak coupling, work in the instantaneous energy eigenbasis of system Hamiltonian:

$$\frac{d}{dt}\rho_S = -i[H_S(t) + H_{LS}(t), \rho_S(t)] + \sum_{\omega} \gamma_{\alpha\beta}(\omega) \left( L_{\omega,\beta}(t)\rho_S L_{\omega,\alpha}^\dagger(t) + \frac{1}{2} \{L_{\omega,\alpha}^\dagger(t)L_{\omega,\beta}(t), \rho_S(t)\} \right)$$

← Unitary evolution with bath-induced Lamb shift

← Non-unitary dissipative dynamics

Thermal transition rate from GS:

$$\frac{d}{dt}\rho_{00}(t) \approx \sum_{i>0} |\mathcal{O}_i(t)|^2 \gamma(\Delta_{i0}) (\rho_{ii}(t) - e^{-\beta\Delta_{i0}} \rho_{00}(t))$$

bath spectral density (FT of bath correlation function)

excitation

relaxation      gap suppresses excitations

matrix element of the system operator (from the system-bath Hamiltonian) in the instantaneous energy eigenbasis

$T_2 = 1/\gamma(0)$  doesn't appear!