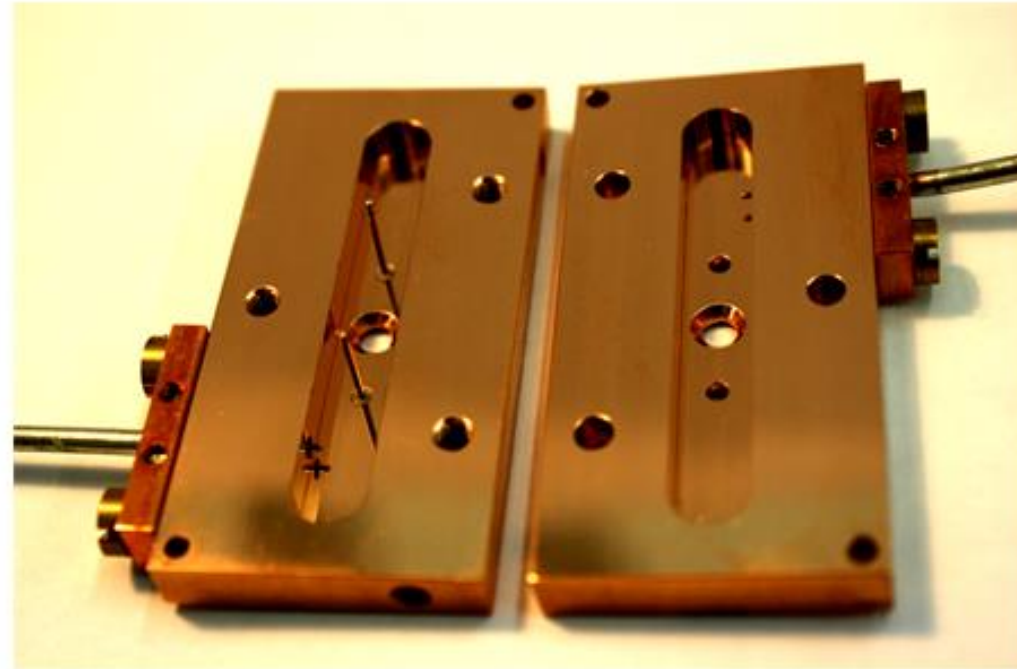


Quantum Device Lab

S. Garcia,
M. Stammeier,
T.Thiele,
[A.Wallraff](#)

Laboratory of Physical
Chemistry

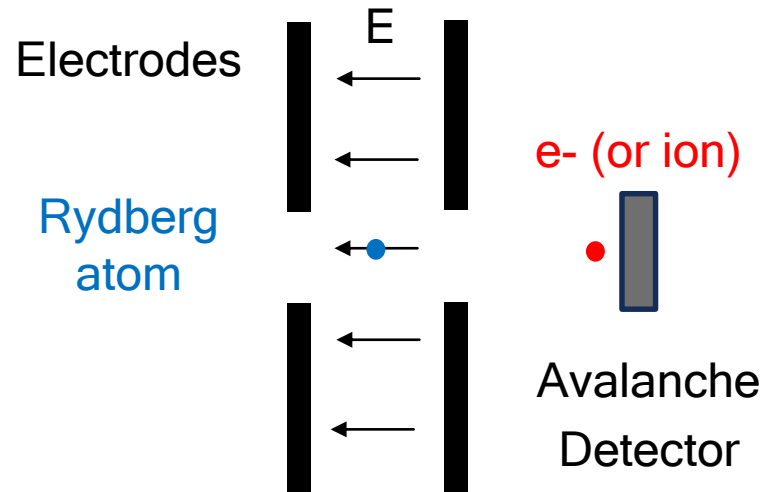
J.Deiglmayr,
J.Agner,
[F.Merkt](#)



Non-destructive detection of ensembles of Rydberg atoms with microwave cavity transmission measurements

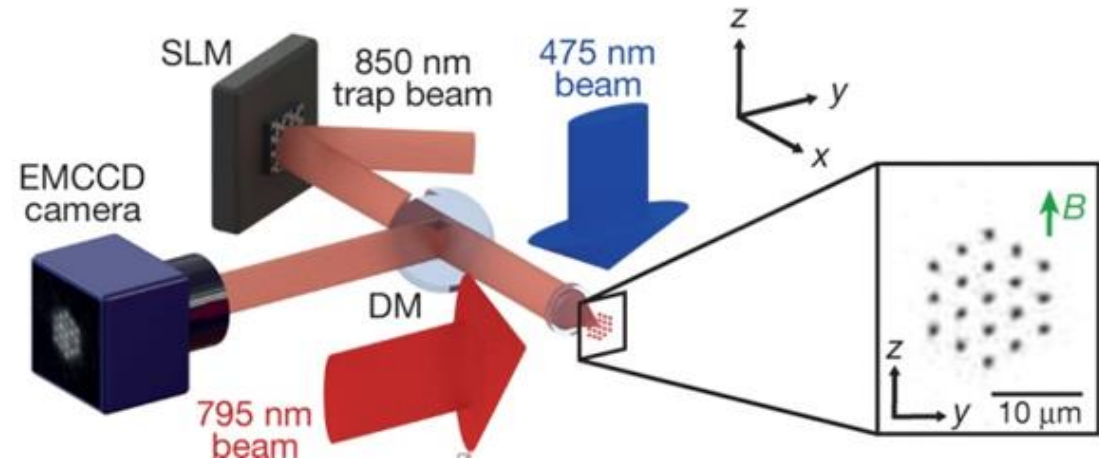
Detection of Rydberg atoms : standard techniques

Ionization



Both methods are very efficient
But they are destructive

Detection of missing atoms



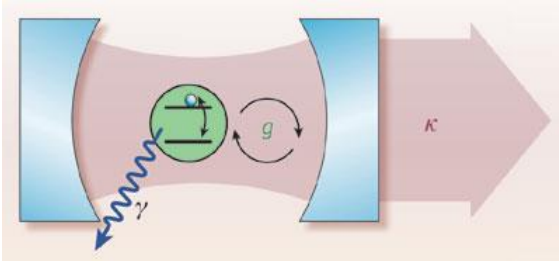
(Labuhn, ... , Browaeys, Nature 534, 2016)

Non destructive detection advantages :

- Multiple measurement with single preparation
- Preparation of quantum states by projection
- Quantum feedback

Cavity QED for non-destructive detection

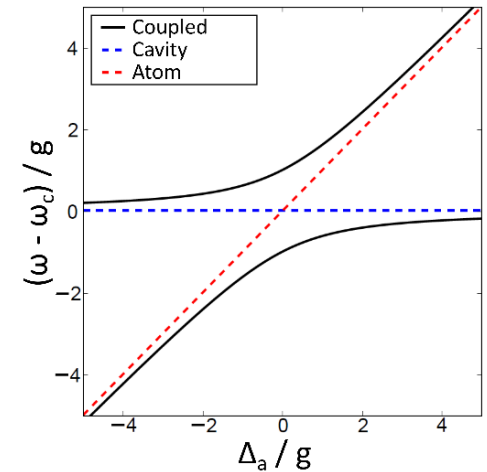
Cavity QED



(Schoelkopf & Girvin, Nature 451, 2008)

$$H_{JC} = \hbar\omega_c(a^\dagger a) + \hbar\frac{\omega_a}{2}\sigma_z + \hbar g(a\sigma_+ + a^\dagger\sigma_-)$$

dispersive limit: $\Delta_a = \omega_a - \omega_c \gg g$



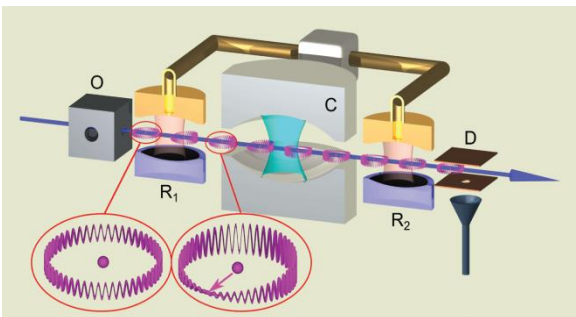
Dispersive Shift - Quantum Non-demolition Measurements

Atomic dispersive shift

$$\delta\omega_a = \frac{g^2}{\Delta_a} (2a^\dagger a + 1)$$

Cavity dispersive shift

$$\chi = \frac{g^2}{\Delta_a} \sigma_z$$

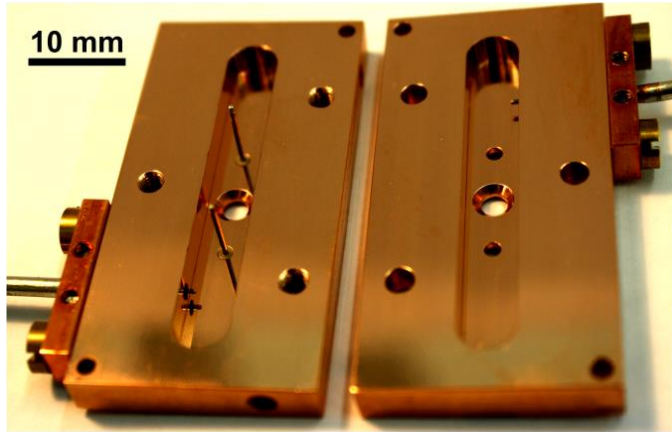


(Haroche, Nobel lecture, 2012)

Cavity QND atomic state measurement:

- Neutral atoms in optical cavity: detection and preparation
- Circuit QED: standard readout
- Rydberg atoms with microwave: ? (Maioli et al., PRL 94, 2005)

Resonator



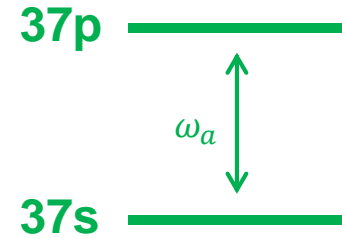
- Rectangular microwave cavity (circuit QED)
- TE₃₀₁ mode: $\frac{\omega_c}{2\pi} = 21.532$ GHz & $\frac{\kappa}{2\pi} = 4$ MHz

Coupling

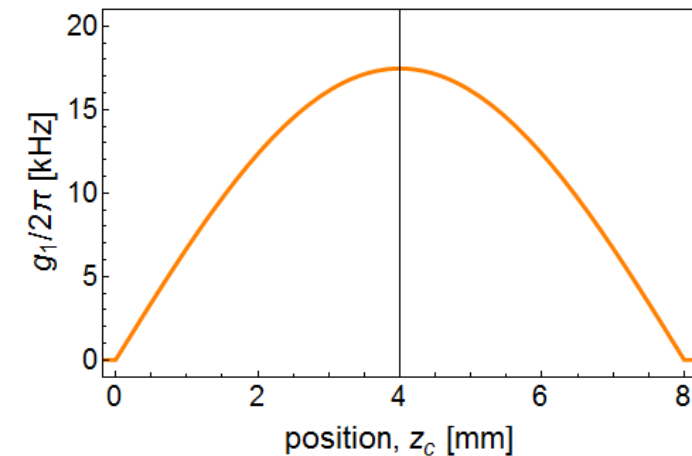
- Coupling $g_1(z) = \frac{d \cdot E_0 \cdot f(z)}{\hbar} \approx 17.5$ kHz at maximum
- Dipole $d = 1092 ea_0$
- Mode function $f(z)$ and $E_0 = 1.2 \frac{\text{mV}}{\text{m}}$
- collective coupling $g_N = g_1 \sqrt{N}$ (Tavis-Cummings)

&

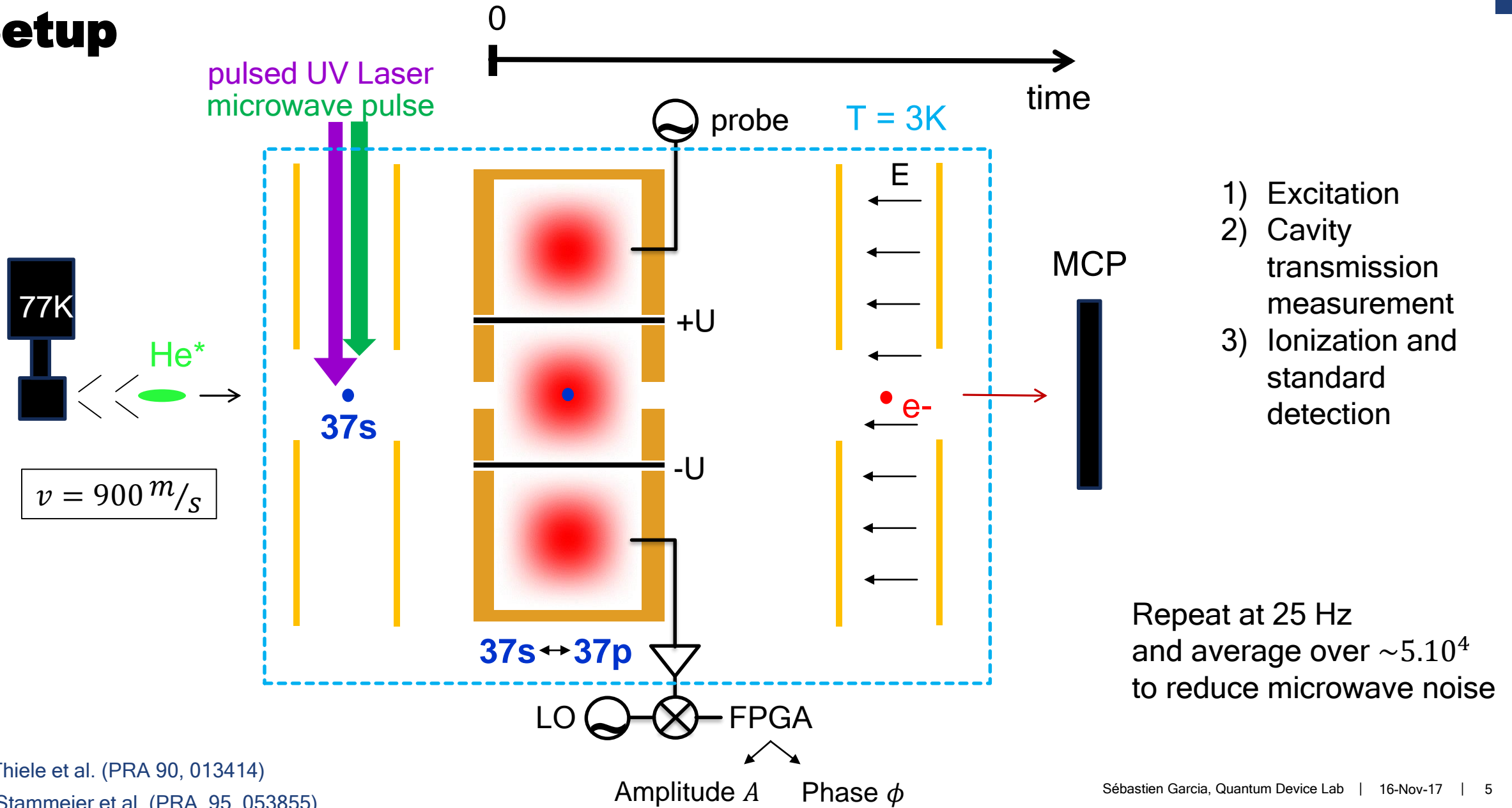
Atom



- Helium atoms
- $\frac{\omega_a}{2\pi} \cong \frac{\omega_c}{2\pi} + 20$ MHz
- (adjusted with DC quadratic Stark effect)

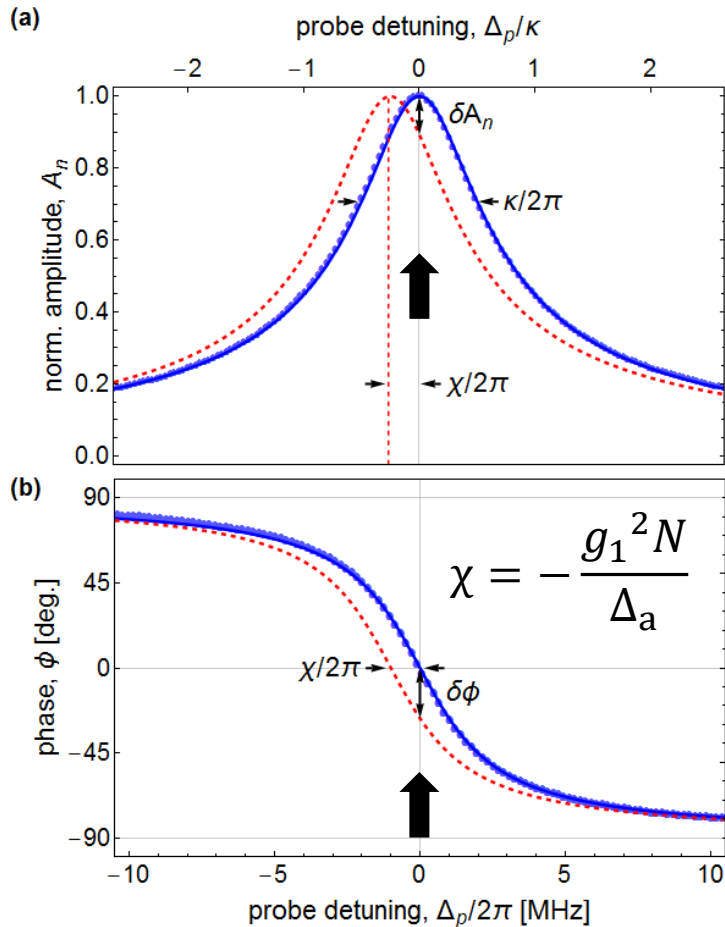


Setup



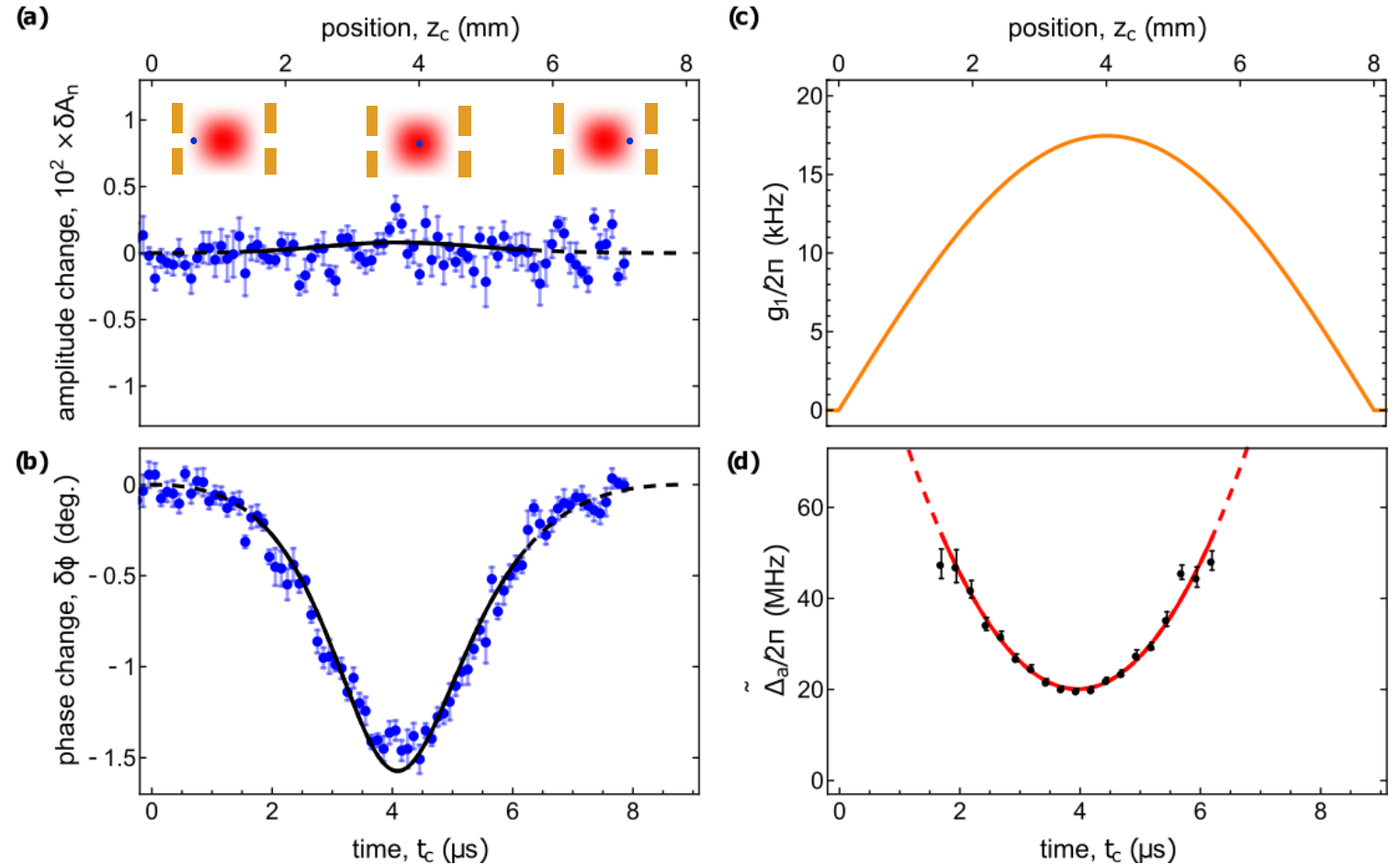
Transmission at cavity resonance

Dispersive effect



Probe detuning: $\Delta_p = \omega_p - \omega_c$
(plot with exaggerated dispersive shift)

Dispersive effect along atomic path

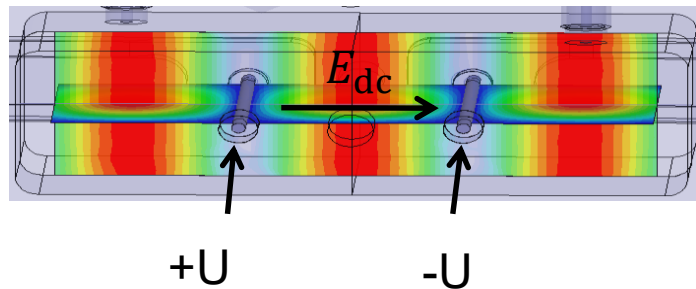


- Fit with coupling and detuning dependence:

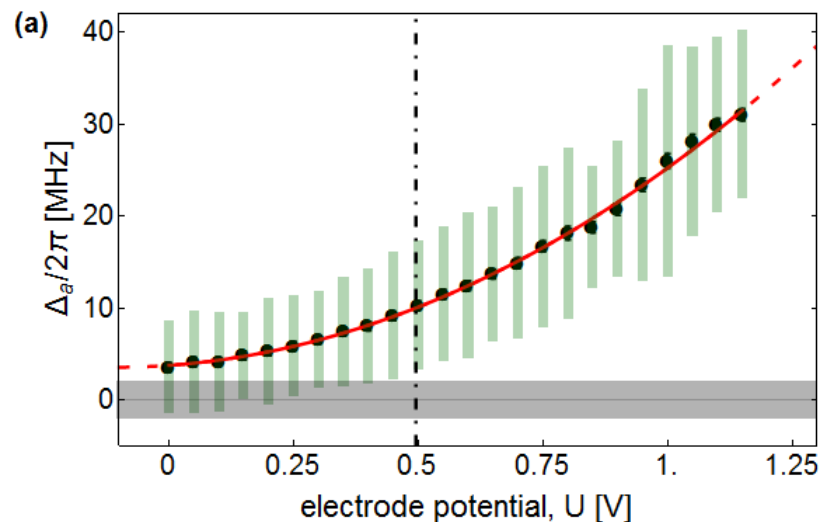
$$N = 3.3(2) \cdot 10^3 \text{ coupled Rydberg atoms}$$

Variation of atom-cavity detuning

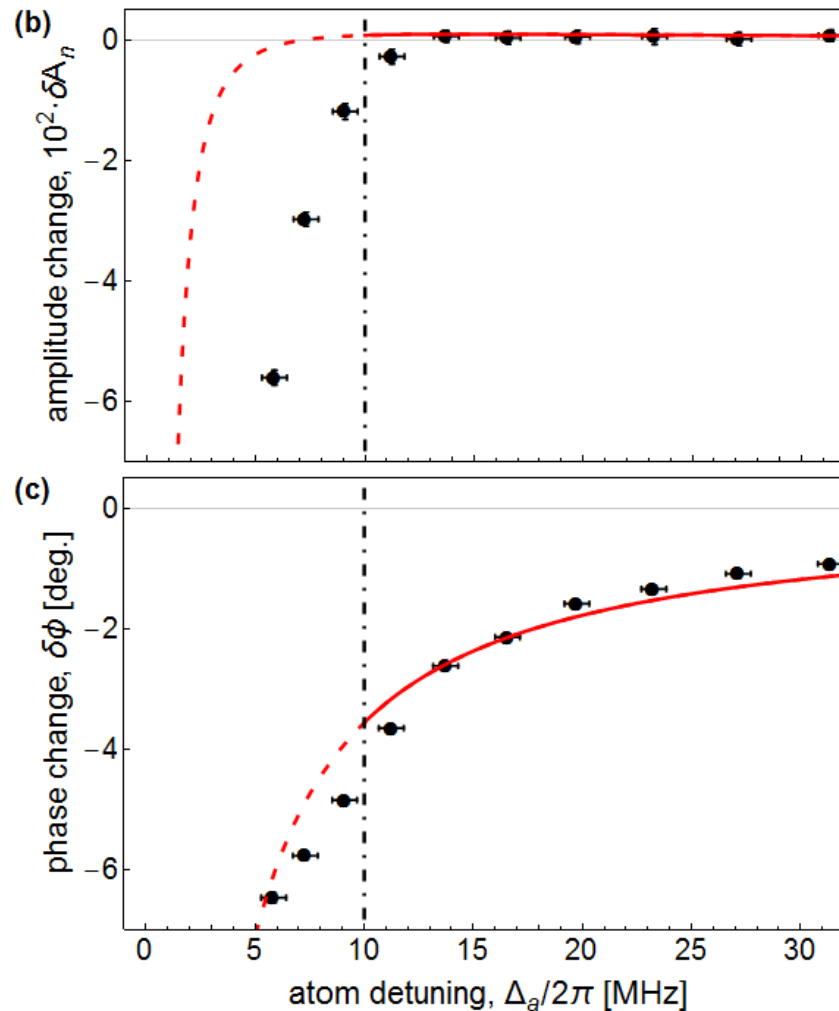
Simulated $|E|$ of the TE301 mode



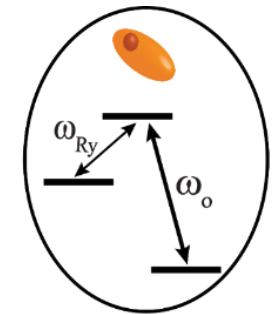
Quadratic Stark effect



Cavity response :



Close to cavity resonance :
atom absorbs a microwave photon and decays to ground state
=> Microwave absorption



Dispersive shift :

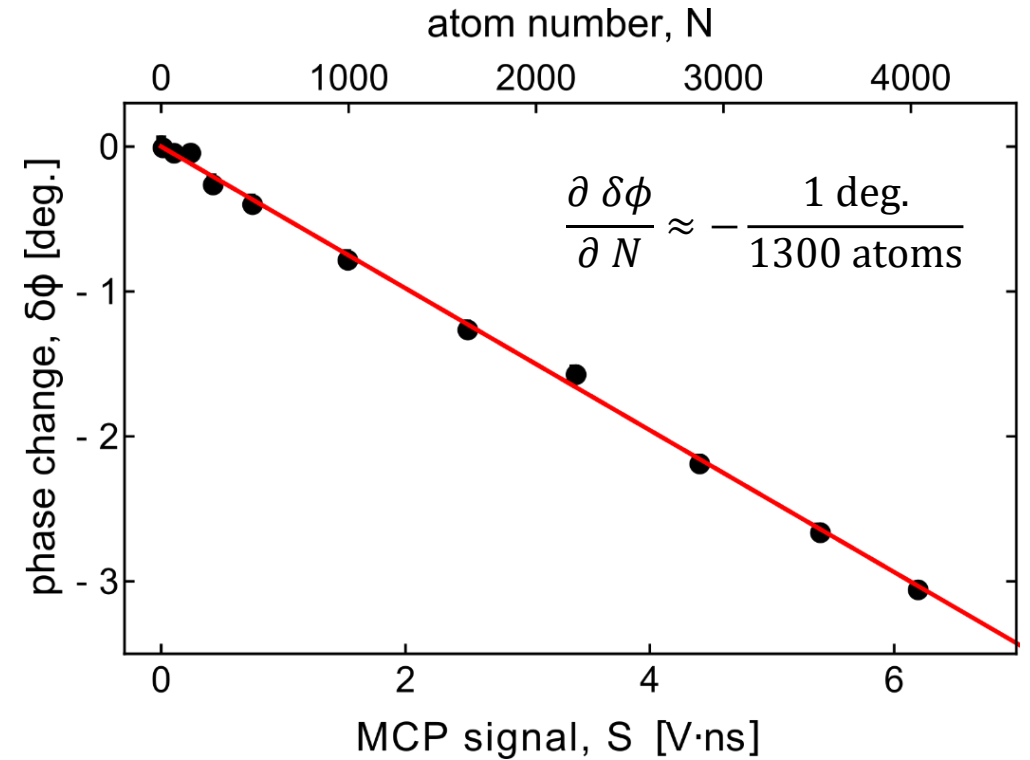
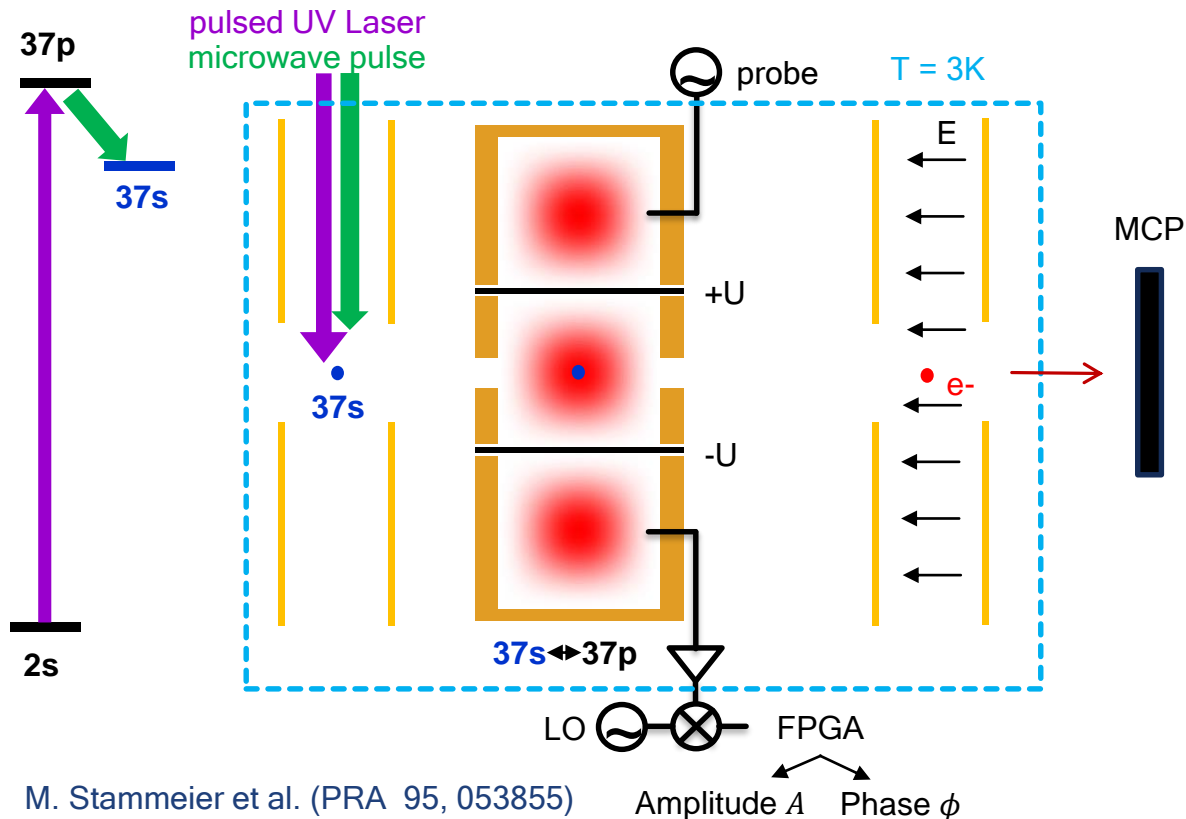
$$\delta\phi \propto \chi = -\frac{g_1^2 N}{\Delta_a}$$

Variation of Rydberg atom number

Measure:

- Phase change (at cavity center)
- Signal on Multi Channel Plate

And change efficiency of **MW transition to 37s state**
 37p atoms decay to ground state before entering cavity



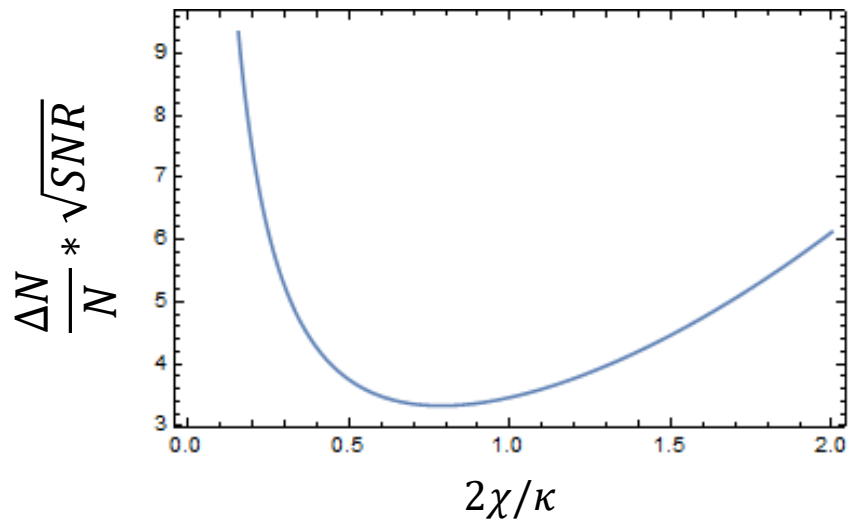
dispersive shift measurement =
 non-destructive detection
 of the number of Rydberg atoms

Detection efficiency

Precision of atom number measurement:

$$\frac{\Delta N}{N} = \left(\frac{2\chi}{\kappa} + \frac{\kappa}{2\chi} \right) \sqrt{2 + \left(\frac{2\chi}{\kappa} \right)^2} \frac{1}{\sqrt{SNR}}$$

Single shot power SNR : $SNR = \frac{n_c \kappa_{out} \tau}{n_{noise}}$



Small dispersive
effect

Lower effect on phase
Lower transmission

Precision optimization :

- 1) Optimal response :
 - ⇒ $g_N \gg \kappa$: collective strong coupling
 - ⇒ Reduce cavity linewidth κ
- 2) Increase interaction time τ :
 - ⇒ Reduce atom velocity
/ Increase cavity length
 - ⇒ Trap atoms
- 3) Optimize output coupling κ_{out} :
 - ⇒ Asymmetric and overcoupled cavity
- 4) Reduce detection noise n_{noise} :
 - ⇒ Quantum limited amplifiers
- 5) Increase number of photons n_c :
 - ⇒ Limited by critical photon number

Critical photon number

- Dispersive shift tends towards 0 for large probe powers due to higher order terms neglected in the dispersive approximation

Single atom (circuit QED):

- Jaynes-Cummings hamiltonian eigenvalues

$$\omega_{\pm, n_c} = n_c \omega_c + \frac{1}{2} (\Delta_a \pm \sqrt{\Delta_a^2 + 4 n_c g_1^2})$$

- Power dependent dispersive shift

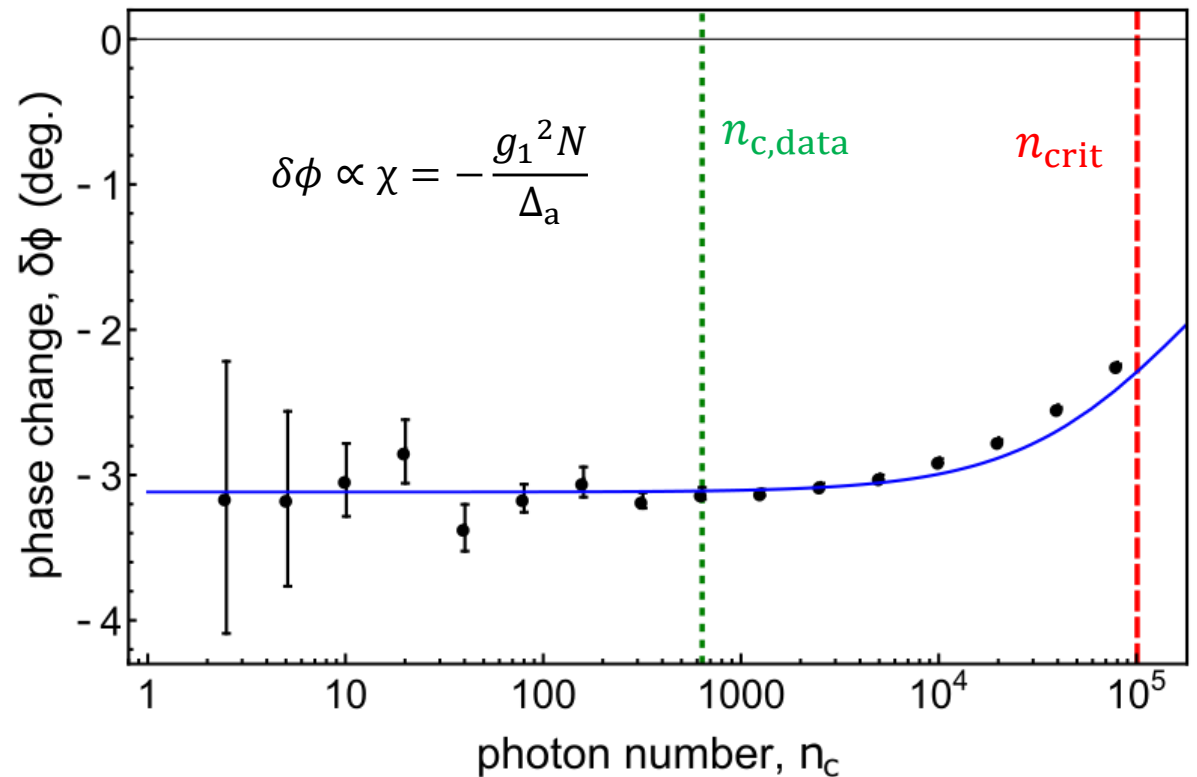
$$\chi(n_c) \simeq -\frac{g_1^2}{\Delta_a} \frac{1}{\sqrt{1 + \frac{n_c}{n_{\text{crit}}}}}$$

$$\text{with } n_{\text{crit}} = \frac{\Delta_a^2}{4 g_1^2}$$

N atoms:

- Tavis-Cummings hamiltonian approx. eigenvalues
- Power dependent dispersive shift

$$\chi(n_c) \simeq -\frac{g_1^2 N}{\Delta_a} \frac{1}{\sqrt{1 + \frac{n_c}{n_{\text{crit}}}}}$$

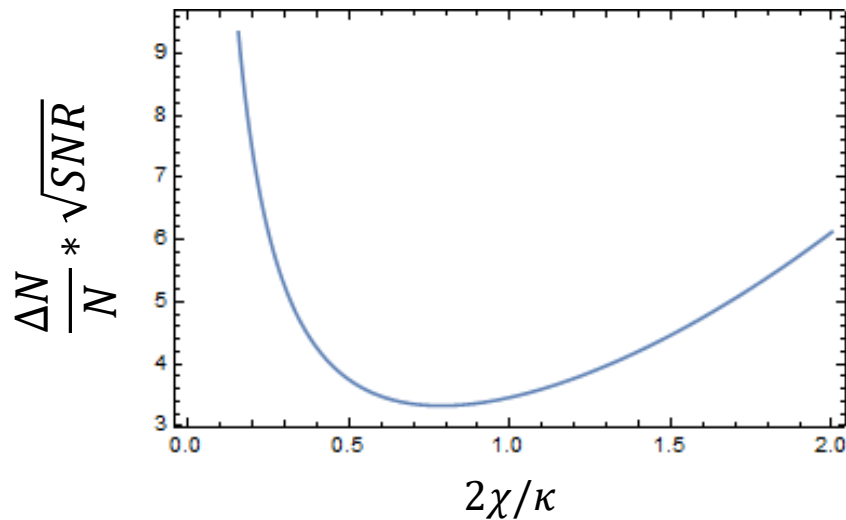


Limit of the detection

Precision of atom number measurement:

$$\frac{\Delta N}{N} = \left(\frac{2\chi}{\kappa} + \frac{\kappa}{2\chi} \right) \sqrt{2 + \left(\frac{2\chi}{\kappa} \right)^2} \frac{1}{\sqrt{SNR}}$$

Single shot power SNR : $SNR = \frac{n_c \kappa_{out} \tau}{n_{noise}}$



Small dispersive effect

Lower effect on phase
Lower transmission

Precision optimization :

1) Optimal response :
 $\Rightarrow g_N \gg \kappa$: collective strong coupling
 \Rightarrow Reduce cavity linewidth κ

2) Increase interaction time τ :
 \Rightarrow Reduce atom velocity
 / Increase cavity length
 \Rightarrow Trap atoms

3) Optimize output coupling κ_{out} :
 \Rightarrow Asymmetric and overcoupled cavity

4) Reduce detection noise n_{noise} :
 \Rightarrow Quantum limited amplifiers

5) Increase number of photons n_c :
 \Rightarrow Limited by critical photon number

With reasonable parameters :

$$N = 4000$$

$$\frac{g_N}{2\pi} = 1.1 \text{ MHz}$$

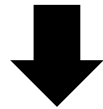
$$\frac{\kappa_{out}}{2\pi} = 300 \text{ kHz}$$

$$\frac{\Delta_{a,opt}}{2\pi} = 10 \text{ MHz}$$

$$\tau = 50 \mu\text{s}$$

$$n_{noise} = 1$$

$$n_c = 880 = 10^{-2} n_{crit}$$



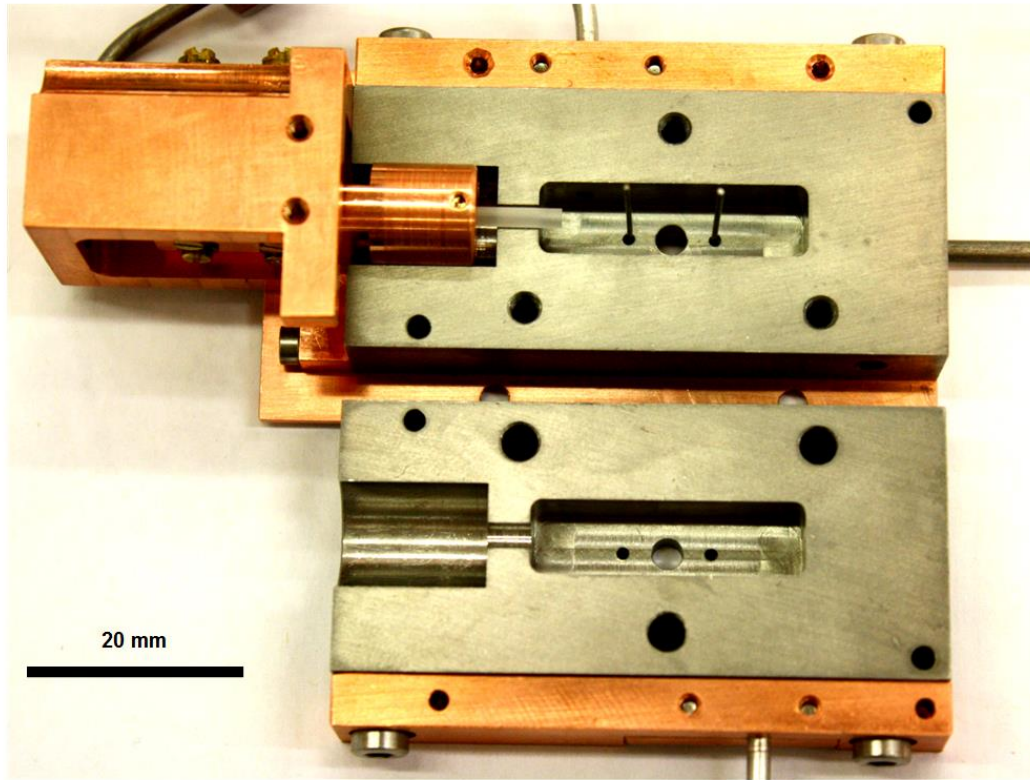
$$\frac{\Delta N}{N} = 1.2 \%$$

$$\Delta N = 49$$

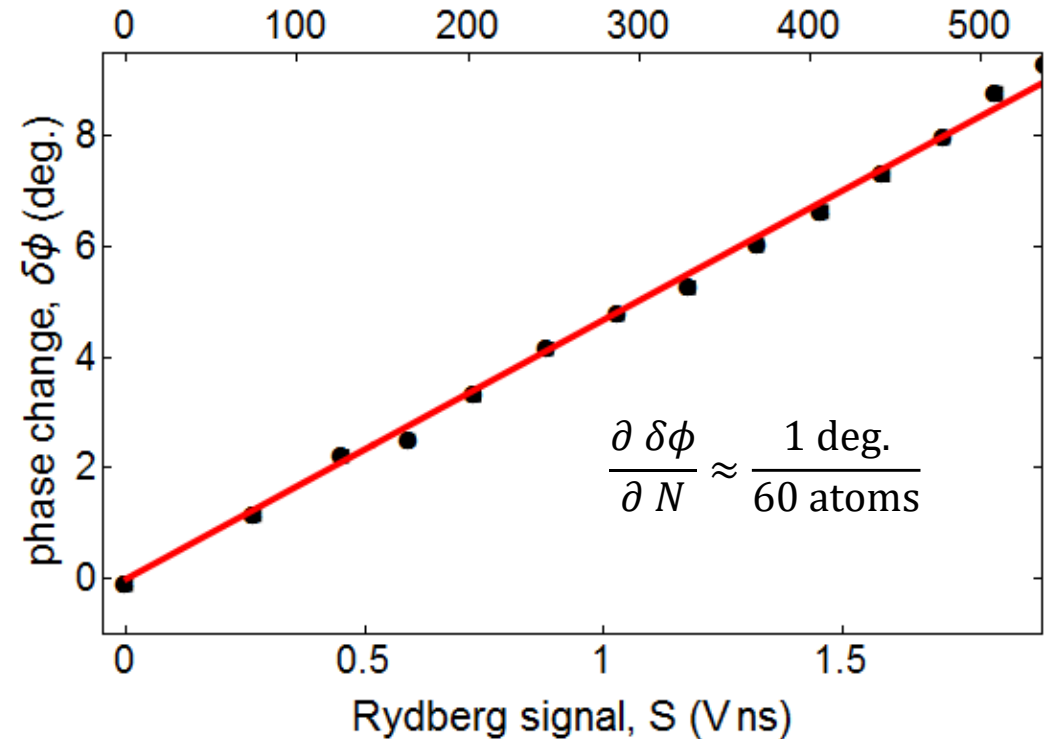
(sub-poissonian)
In a single-shot measurement

Improvement of the detection (1st step)

Superconducting cavity (Niobium):



Atom number measurement:



Gain of a factor ≈ 20 in sensitivity

$$\frac{\kappa_{int}}{2\pi} = 12 \text{ kHz}$$

$$\frac{\kappa}{2\pi} = 237 \text{ kHz}$$

Conclusions

- Transmission measurement of dispersive shift induced by Rydberg atoms
- Non-destructive detection of atom numbers :
Possible application : Merged beam experiments (P. Allmendinger et al., ChemPhysChem 17, 3596 (2016))

Outlook

- Measurement of pseudo-spin $J_z = \frac{1}{2} \sum_j \sigma_{z,j}$ of the ensemble $\chi = \frac{g N^2}{\Delta_a} \frac{J_z}{N/2}$
- Increase the coupling by using photons confined in 2D waveguides
- Hybrid cavity QED
with superconducting qubit

